Delhi Public School, Howrah

PERIODIC TEST-I (2024 -2025)

Class-XI

Care must be taken not to write anything on the question paper. All the questions must be attempted in the correct sequence.

Mathematics (041)

Time: 3 Hours **F.M. 80**

General Instructions:

This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

Section –A
(Multiple Choice Questions)
Each question carries 1 mark

1) The set builder form of the null set is
   a) $\{x : x = x\}$ b) $\emptyset$ c) $\{x : x \neq x\}$ d) { }
2) For any two sets A and B, the value of $A \cup (A \cap B)$ is
   a) A b) B c) $\emptyset$ d) none of these
3) Let S = set of all points inside a square, T = the set of points inside the triangle and C = the set of points inside the circle. If the triangle and circle intersect each other and are contained in the square, thenwhich of the following is true?
   a) $S \cap T \cap C = \emptyset$ b) $S \cup T \cup C = C$ c) $S \cup T \cup C = S$ d) $S \cup T = S \cap C$
4) Given U is [- 5, 5] and A is (- 3, 5], then A' is
   a) [-5, -3) b) (4, 5] c)[4, 5] d) [-5, -3]
5) If A and B are two sets and $A \subset B$, then
   a) n(A) > n(B) b) n(A) < n(B) c)n(A) ≥ n(B) d) n(A) ≤ n(B)
6) If R = {(1,3), (2, 5), (3, 5), (7, 4)} is a relation then the number of elements in its range is
   a) 2 b) 3 c) 4 d) 8
7) If (2x, y – x) = (y + 3, 0), then y is equal to
   a) 3 b) - 3 c) x d) – x
8) The range of the real function $f(x) = 1 + 3\cos\ 2x$is
   a) [- 4, 2] b) $[4, 2]$ c) [-2, 4] d) [2, - 4]
9) Let n(A) = m and n(B) = n. Then the total number of non – empty relations that can be defined from A to B is
   a) m^n b) $n^m - 1$ c) nm - 1 d) $2^{mn} - 1$
10) The value of i^{257} is
    a) – 1 b) i c) 1 d) – i
11) Conjugate of the complex number $i^3 - 4$ is
    a) $i^3 + 4$ b) 4 – i c) – 4 + i d) – 4 – i
12) If z_1 and z_2 are two complex numbers, then $|z_1z_2|$ is equal to
    a) z_1z_2 b)$\overline{z_1}\,\overline{z_2}$ c) $|z_1||z_2|$ d) $\sqrt{z_1z_2}$

13) Angle formed by the large hand of a clock in 20 minutes is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{3\pi}{4}$ d) $\frac{2\pi}{3}$

14) If $\sin\theta + cosec\,\theta = 2$, then $sin^2\theta + cosec^2\,\theta$ is equal to

a) 1 b)2 c) 3 d) 4

15) $\tan(\frac{3\pi}{2} + \theta)$ is equal to

a) $-\tan\theta$ b) $\tan\theta$ c) $\cot\theta$ d) $-\cot\theta$

16) If tan A = ½ and tan B = 1/3, then tan (2A + B) is

a) 4 b) 3 c) 2 d) 1

17) The value of sin 50^o – sin 70^o + sin 10^o is

a) 1 b) 0 c) 1/2 d) 2

18) The minimum value of 3 cos x + 4 sin x + 8 is

a) 5 b) 9 c) 7 d) 3

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

19) Assertion(A): The equation ix^2 -3ix + 2i = 0 has non real roots.

Reason (R): If a, b, c are real and $b^2 - 4ac > 0$, then the roots of the equation $ax^2 + bx + c = 0$ are real and if $b^2 - 4ac < 0$, then the roots of the equation $ax^2 + bx + c = 0$ are not real.

20) Assertion(A): The ratio of radii of two circles at the centres of which two equal arcs subtend angles 30^o and 70^o is 21:10.

Reason (R):Number of radians in an angle subtended at the centre of a circle by an arc is equal to the ratio of the length of the arc to the radius of the circle.

Section –B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21) a) State whether the given statement is true or false. If it is true, prove it and if it is false, give an example. If $A \subset B$ and $B \in C$, then $A \in C$.

OR

b) Show that if $A \cup B = A \cap B$, it implies that A = B.

22) a) Find the range of $f(x) = 1 - |x - 2|$.

OR

b)Let $f(x) = \begin{cases} 3 - 2x, & x < 1 \\ 3x + 5, & 1 \le x \le 2 \\ 5x - 2, & x > 2 \end{cases}$, then find 2 f(0) + f(3).

23) Prove that $(1 + cot\theta - cosec\,\theta)(1 + \tan\theta + \sec\theta) = 2$.

24) Find the degree measure of $\frac{\pi}{64}$ rad.

25) Express $(-\sqrt{3} + \sqrt{-2})(-2 + \sqrt{-3})$ in the form a+ib.

Section – C

[This section comprises of short answer type questions (SA) of 3 marks each]

26) If A = set of letters in the word 'JAIPUR' and B = set of letters of the word 'JODHPUR', find
i) AUB ii) B – A . Also verify $n\ (A \cup B) = n\ (A) + n\ (B) - n(\ A \cap B)$.

27) a) Find the number of non – zero integral solutions of the equation $\left|3^{\frac{1}{2}} - i\right|^{x} = 4^{x}$.

OR

b) Find real θ such that $\frac{3+2i\ sin\theta}{1-2i\ sin\theta}$ is purely real.

28) Write the domain and range of greatest integer function and draw its graph.

29) Find the domain and range of the function $f$defined by $f(x) = \sqrt{25 - x^2}$.

30) If $\sin\alpha = k\sin\beta$,prove that $\tan\left(\frac{\alpha-\beta}{2}\right) = \frac{k-1}{k+1}\tan(\frac{\alpha+\beta}{2})$.

31) a) Find the value of $\tan\frac{13\pi}{12}$.

OR

b) Find the value of $\sin 22\frac{1}{2}^{o}$.

Section –D

[This section comprises of long answer type questions (LA) of 5 marks each]

32) a) If $cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$, then find the value of $\cos 2\alpha + \cos 2\beta$.

OR

b) Prove that: $\frac{\sin x}{\cos 3x} + \frac{\sin 3x}{\cos 9x} + \frac{\sin 9x}{\cos 27x} = \frac{1}{2}\ (\tan 27x - \tan x)$.

33) Let U = { x : x ∈ N and x ≤ 8}, A = { x : 5 < x^2< 50} and B = { x : x is prime}. Draw a Venn Diagram to show the relationship between the given sets. Hence list the elements of the following sets:
i) A' ii) B' iii) A – B. Is A – B = A ∩ B'?

34) i) If f (x) = y = $\frac{ax-b}{cx-a}$, then prove that f (y) = x. 2
ii) IfA={x:x∈ W,x < 2}, B= {x: x∈N, 1 <x < 5}, C= {3, 5}, thenfind
(a) Ax(B∩C) (b) Ax(B∪C) 3

35) a) If $z = x + iy$ and imaginary part of$\frac{2z+1}{iz+1}$ is – 2, then show that x + 2y – 2 = 0.

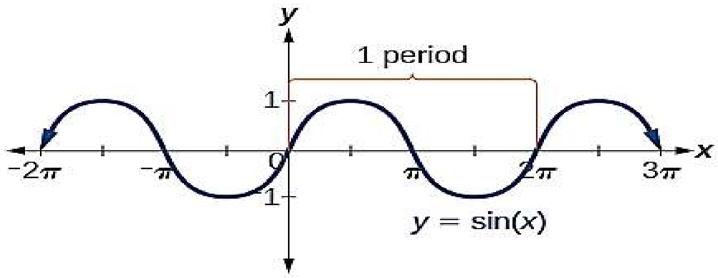
OR

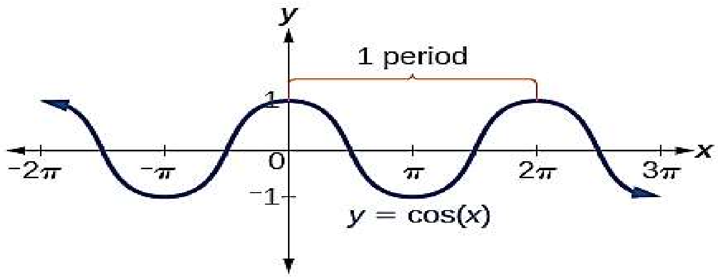
b) i) If $(x + iy)^3$ = u + iv, then show that $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$. 3
ii) If $(\frac{1-i}{1+i})^{100} = a + ib$, then find(a, b). 2

Section –E

[This section comprises of 3 case- study/passage-based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.)

36) **CASE STUDY 1:**Two brothers are plying their bicycles over two curved pathways. Their teacher tells them that one of the brothers is following the path of curve y= sin x while, the other one is following the path of curve y = cos x. Refer to the graphs of sine and cosine function as given below.





Based on this information answer the following questions

i) Write the domain and range of Sine function. 1

ii) a) If $\cos x = -\frac{3}{5}$ and x lies in the 3rd quadrant, then find the value of tan x. 1

OR

b) Find the value of $\sin\frac{31\pi}{3}$. 1

iii) Evaluate : sin (45° + θ) – cos (45° – θ). 2

37) **CASE STUDY 2:** A relation R from a non-empty set A to a non-empty set B is a subset of the cartesian product set A × B. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in A × B.

The set of all first elements in a relation R is called the domain of the relation R, and the set of all second elements called images is called the range of R.

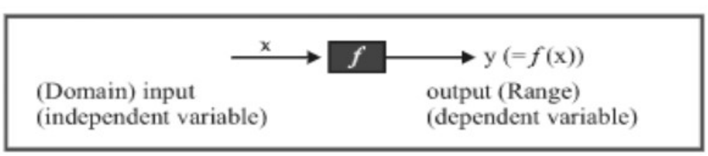
A relation may be represented either by the Roster form or by the set of builder form, or by an arrow diagram which is a visual representation of relation.

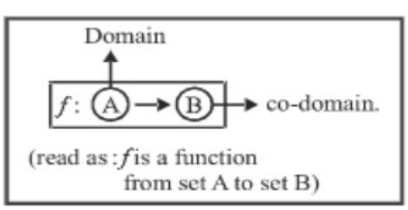
If n(A) = m, n(B) = n, then n(A × B) = mn and the total number of possible relations from set A to set B = 2^{mn}

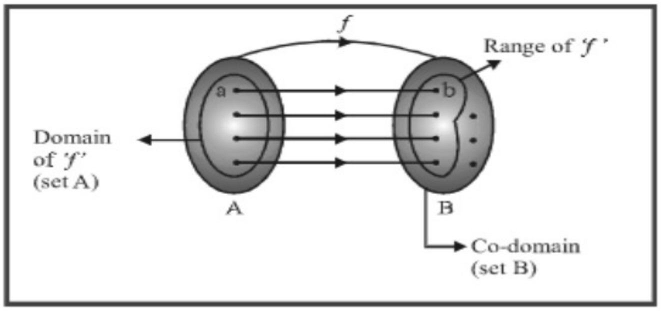
A relation 'f' is said to be a function, if every element of a non-empty set X, has only one image or range to a non-empty set Y.

OR

If 'f' is the function from X to Y and (x,y) ∈ f, then f(x) = y, where y is the image of x, under function f and x is the preimage of y, under 'f'. It is denoted as;f: X → Y.







Based on the given source answer the following:

i) If $f(x) = x^2 + 2$, $x \in R$, then find the range of f(x). 1

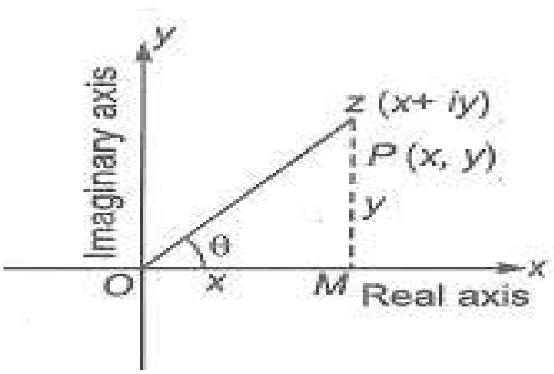
ii) What will be the domain for which the functions $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are equal? 1

iii) a) Find the domain and the range of the function $f(x) = \frac{x-2}{3-x}$. 2

OR

b) If f (x) = ax + b, where a and b are integers, f (–1) = – 5 and f (3) = 3, then find a and b. 2

38) CASE STUDY 3: Any complex number z=x+iy can be represented geometrically by a point (x, y) on a plane, called Argand plane or Gaussian plane. The angle made by the line joining the point z to the originwith the x-axis is called argument of that complex number. It is denoted by the symbol arg (z) or amp (z).



Argument (z) = $\theta = \tan^{-1}(y/x) \Longrightarrow \tan\theta = y/x$

A purely real number is represented by a point on x-axis. A purely imaginary number is represented by a point on y-axis. There exists a one-one correspondence between the points of the plane and the members of the set C of all complex numbers.

The length of the line segment OP is called the modulus of z and is denoted by |z| ,i.e., length of OP=$\sqrt{x^2 + y^2}$.

If z = x + iy be a complex number, then a complex number obtained by changing the sign of imaginary part of the complex number is called the conjugate of z and it is denoted by $\bar{z}$, i.e., $\bar{z}$ = x – iy. Note that additive inverse of z is – x– iy but conjugate of z is x – iy.

Based on the given information answer the following questions:

i) If $\frac{z-1}{z+1}$ is purely imaginary number($z \neq -1$), then find the value of |z|. 2

ii) If $\alpha > 0$ and $z = \frac{(1+i)^2}{\alpha - i}$, has magnitude $\sqrt{\frac{2}{5}}$, then find $\bar{z}$. 2