



Delhi Public School, Howrah

PERIODIC TEST-I (2024-2025)

Class-XII

Care must be taken not to write anything on the question paper. All the questions must be attempted in the correct sequence.

Subject: - Mathematics (Code No-041)

Time: 3 Hours

F.M. 80

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

Section- A

(Multiple Choice Questions)

Each question carries 1 mark

1. If $\tan^{-1} x = \sin^{-1} \left(\frac{1}{2} \right)$, then the value of x is
a) 1 b) $\sqrt{3}$ c) $\frac{1}{\sqrt{3}}$ d) -1
2. Let $f(x) = \begin{cases} \frac{\log(5+x) - \log(5-x)}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x=0$. The value of k is
a) $\frac{1}{5}$ b) $\frac{2}{5}$ c) $\frac{3}{5}$ d) $\frac{4}{5}$
3. If area of the triangle with vertices (1,-1), (-4,k) and (-3,-5) is 24 sq. units (where $k>0$), then the value of k is
a) 18 b) 28 c) 6 d) 12
4. If $A = \begin{bmatrix} 5 & x^2 \\ 3x-2 & x-1 \end{bmatrix}$ is a symmetric matrix, then the value(s) of x are:
a) $x=-1, -2$ b) $x=1, -2$ c) $x=-1, 2$ d) $x=1, 2$
5. Let A be a non-singular matrix of order 3 and $|\text{adj } A| = |A|^k$, then the value of k is
a) 1 b) 2 c) 3 d) 4
6. A relation R in the set $A = \{1,2,3\}$ is defined as $R = \{(1, 1), (1, 2), (2, 2), (3, 3)\}$. Which of the following ordered pair in R shall be removed to make it an equivalence relation in A?
a) (1,1) b) (1,2) c) (2,2) d) (3,3)
7. If A and B are square matrices of same order such that $AB=A$ and $BA=B$, then A^2+B^2 is equal to
a) AB b) A+B c) 2AB d) 2BA
8. The domain of the function $\sin^{-1}(x^2 - 4)$ is
a) [3,5] b) $\left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$ c) $[-\sqrt{5}, -\sqrt{3}] \cap [\sqrt{3}, \sqrt{5}]$ d) $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$
9. Which of the following relations on I (set of all integers) is not an equivalence relation?
a) $(a, b) \in R \Leftrightarrow a + b$ is even
b) $(a, b) \in R \Leftrightarrow a = b$
c) $(a, b) \in R \Leftrightarrow a - b$ is even
d) $(a, b) \in R \Leftrightarrow a < b$

10. If A is a square matrix such that $A^2 = I$, then $(I + A)^3 + (A - I)^3 - 7A$ is equal to
 a) A b) $A - I$ c) $A + I$ d) $3A$
11. If $x = \sqrt{a^{\sin^{-1} t}}$, $y = \sqrt{a^{\cos^{-1} t}}$ ($a > 0$ and $t \in (-1, 1)$), then $\frac{dy}{dx}$ is equal to
 a) $-\frac{y}{x}$ b) $\frac{y}{x}$ c) $\frac{x}{y}$ d) $-\frac{x}{y}$
12. If $y = (\log x)^2$, then $\frac{d^2y}{dx^2}$ at $x = 1$ is
 a) 0 b) 1 c) 2 d) not defined
13. Let A be a non-singular matrix such that $3A^2 - 2A + 4I = O$, then A^{-1} in terms of A is
 a) $\frac{1}{4}(2I + 3A)$ b) $\frac{1}{4}(I - 3A)$ c) $\frac{1}{4}(3A - I)$ d) $\frac{1}{4}(2I - 3A)$
14. If A and B are non-singular matrices of same order, then choose the correct option:
 a) AB is non-singular always
 b) AB is singular always
 c) $(AB)^{-1} = A^{-1}B^{-1}$
 d) AB is not invertible
15. Let $f(x) = \begin{cases} ax^2 - b, & x < 1 \\ \frac{1}{x^2}, & x \geq 1 \end{cases}$ is differentiable at $x=1$. Then the values of a and b are
 a) $a=1, b=2$ b) $a=-1, b=-2$ c) $a=1, b=-2$ d) $a=-1, b=2$
16. Let $f: N \rightarrow N$ be defined as $f(x) = x^2 + x + 1$, then f is
 a) One-one but not onto
 b) Onto but not one-one
 c) One-one and onto both
 d) Neither one-one nor onto
17. If $x = a \sin t$, $y = a(\cos t + \log(\tan \frac{t}{2}))$, then $\frac{dy}{dx}$ is equal to
 a) $\tan t$ b) $\cot t$ c) $\sec t$ d) $\operatorname{cosec} t$
18. If $y = 5 \cos x - 3 \sin x$, then $\frac{d^2y}{dx^2}$ is equal to
 a) y b) -y c) $\frac{1}{y}$ d) $-\frac{1}{y}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

- (a) Both (A) and (R) are true and (R) is the correct explanation of (A).
 (b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
 (c) (A) is true but (R) is false.
 (d) (A) is false but (R) is true.

19. Assertion(A): If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then $x = \pm 6$

Reason(R): If A is a skew-symmetric matrix of odd order, then $|A|=0$.

20. Assertion(A): Minor of an element of a determinant of order $n(n \geq 2)$ is a determinant of order n .

Reason(R): If A is an invertible matrix of order 2, then $\det(A^{-1})$ is equal to $1/\det A$.

Section-B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21. (a) Find the principal value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

OR

(b) Evaluate: $\sin\left(\cot^{-1}\left(\cot\frac{17\pi}{3}\right)\right)$

22. (a) If $\cos y = x \cos(a + y)$, prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$

OR

(b) Find $\frac{dy}{dx}$, when $y = \sin^{-1}\left(\frac{3 \sin x + 4 \cos x}{5}\right)$

23. If $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$, find α satisfying $0 < \alpha < \frac{\pi}{2}$ when $A + A^T = \sqrt{2} I_2$, where A^T is the transpose of A.

24. If the function $f(x)$ defined by $f(x) = \begin{cases} \frac{1 - \cos cx}{x \sin x}, & x \neq 0 \\ \frac{1}{2}, & x = 0 \end{cases}$ is continuous at $x=0$, find the value(s) of c .

25. Is the given system of equations consistent? Give reason.

$$x + 3y = 5, 2x + 6y = 8$$

Section-C

[This section comprises of short answer type questions (SA) of 3 marks each]

26. Let N be the set of all natural numbers and R be the relation on $N \times N$ defined by

$(a, b)R(c, d)$ iff $ad(b + c) = bc(a + d)$ for all $a, b, c, d \in N$. Show that R is an equivalence relation.

27. (a) Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, $x \neq 0$.

OR

(b) If $f(x) = \left(\frac{3+x}{1+x}\right)^{2+3x}$, find $f'(0)$.

28. Express the matrix $\begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrix.

29. Find the value of k for which the function f defined by

$$f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & \text{if } -1 \leq x < 0 \\ \frac{2x+1}{x-1}, & \text{if } 0 \leq x < 1 \end{cases}$$

is continuous at $x=0$.

30. (a) If $y = \sin(m \sin^{-1} x)$ prove that $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + m^2y = 0$

OR

(b) If $x = \sin t, y = \sin pt$, prove that $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + p^2y = 0$

31. (a) For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ show that $A^2 - 5A + 4I = O$. Hence find A^{-1} .

OR

(b) For the matrix $A = \begin{bmatrix} 2 & 4 \\ 1 & -3 \end{bmatrix}$, verify $(A^{-1})' = (A')^{-1}$

Section-D

[This section comprises of long answer type questions (LA) of 5 marks each]

32. (a) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} . Using A^{-1} , solve the following system of linear equations:

$$x + 2y + z = 4, -x + y + z = 0, x - 3y + z = 2$$

OR

(b) Use the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations:

$$x - y + 2z = 1, 2y - 3z = 1, 3x - 2y + 4z = 2.$$

33. (a) Let $A = \{1, 2, 3, \dots, 9\}$ and R be the relation defined on $A \times A$ by $(a, b)R(c, d)$ iff $a + d = b + c$. Prove that R is an equivalence relation. Also find the equivalence class $[(2, 5)]$.

OR

(b) Show that the function f defined by $f(n) = \begin{cases} n + 1, & \text{if } n \text{ is even} \\ n - 1, & \text{if } n \text{ is odd} \end{cases}$ is bijective.

34. Show that the function $f(x) = |x - 1| + |x + 1|$, for all $x \in \mathbb{R}$ is not differentiable at the points $x = -1$ and $x = 1$.

35. Find the inverse of the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and hence show that $A^{-1}A = I$

Section-E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.]

36. Case Study-1:

On his birthday, Ishan decided to donate some money to children of an orphanage home. If there were 8 children less, every one would have got Rs 10 more. However, if there were 16 children more, every one would have got Rs 10 less. Let the number of children be x and amount distributed by Ishan to each child be Rs y .



Based on the information given above, answer the following questions:

- (i) Frame the equations in x and y which represent the above situation. 1
- (ii) Write the matrix form of the equations obtained from question (i). 1
- (iii) (a) If A is the coefficient matrix of the above situation, find A(adj A). 2

OR

- (b) Find the number of children and the amount distributed by Ishan to each child. 2

37. Case Study-2:

A function f is said to be continuous at x=a iff

- (i) $\lim_{x \rightarrow a^-} f(x)$ exists
- (ii) $\lim_{x \rightarrow a^+} f(x)$ exists and
- (iii) $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$.

$$\text{Let } f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, & \text{if } x < 4 \\ a + b, & \text{if } x = 4 \\ \frac{x-4}{|x-4|} + b, & \text{if } x > 4 \end{cases}$$

Based on the information given above, answer the following questions :

- I. Find the value of $\lim_{x \rightarrow 4^-} f(x)$ 1
- II. Find the value of $\lim_{x \rightarrow 4^+} f(x)$ 1
- III. If f(x) is continuous at x=4, then find the values of a and b 2

38. Case Study-3:

A function f(x) is said to be differentiable at x=c if

- (i) Left hand derivative (L.H.D.)= $\lim_{h \rightarrow 0^-} \frac{f(c+h)-f(c)}{h}$ exists finitely
- (ii) Right hand derivative (R.H.D.)= $\lim_{h \rightarrow 0^+} \frac{f(c+h)-f(c)}{h}$ exists finitely and
- (iii) L.H.D.=R.H.D. at x=c.

$$\text{For the function } f(x) = \begin{cases} |x - 3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$$

based on the above information, answer the following questions:

- I. What is RHD of f(x) at x=1? 2
- II. What is LHD of f(x) at x=1? 2