Delhi Public School,Howrah

PERIODIC TEST - I (2024 - 2025)

Class - XII

Care must be taken not to write anything on the question paper. All the questions must be attempted in the correct sequence.

Subject: - Applied Mathematics (Code No-241)

Time: 3 Hours **F.M.80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section iscompulsory. However, there is some internal choice in some questions.
2. Section A has 18 MCQ's and 02 Assertion Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) questions of 2 marks each.
4. Section C has 6 Short Answer (SA) questions of 3 marks each.
5. Section D has 4 Long Answer (LA) questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units ofassessment (04 marks each) with sub parts.
7. Internal Choice is provided in 2 questions in Section-B, 2 questions in Section-C, 2 Questions in Section-D. You have to attempt only one alternative in all such questions.

SECTION A

(All Questions are compulsory. No internal choice is provided in this section)

1) If $x \in R, |2x+3| < 7$,then the solutions of x : 1

(a) $x \in (-5,2)$ (b) $x \in (-5,9]$ (c) $x \in (-\infty,-5) \cup (2,\infty)$ (d) None of these

2) If $x \equiv 4(mod\ 7)$,then the least positive values of x is 1

(a) {4, 11, 18, …} (b) {11, 18, 25, …} (c) {4, 8, 12, …} (d) {1, 8, 15, …}

3) Thetangents to the curve $y^2 + x^2 = 2$at the points (1,1) and (-1,1) are 1

(a) parallel (b) at right angles

(c) neither parallel nor at right angles (d) none of these

4) If the matrix $A = \begin{bmatrix} 0 & -1 & 3x \\ 1 & y & -5 \\ -6 & 5 & 0 \end{bmatrix}$ is skew - symmetric, then the values of x and y are 1

(a) $x = -2, y = 0$ (b) $x = -2, y = 1$

(c) $x = 2, y = 0$ (d) None of these

5) A spherical ice ball is melting at the rate of 100π cm^3/min. The rate (in cm/min) at which its radius is decreasing, when its radius is 15 cm, is 1

(a) $\frac{1}{9}$ (b)$\frac{1}{9\pi}$ (c)$\frac{1}{18}$ (d) $\frac{1}{36}$

6) The value of $\begin{vmatrix} 1+a & 0 & 0 \\ 0 & 1+b & 0 \\ 0 & 0 & 1+c \end{vmatrix}$ is 1

(a) abc (b) $a+b+c$ (c) $(1+a)(1+b)(1+c)$ (d) 0

7) If A and B are symmetric matrices of same order, then AB – BA is a 1

(a) symmetric matrix (b) skew – symmetric matrix

(c) zero matrix (d) identity matrix

8) If $f(x) = x^{x^{x^{\dots\infty}}}$ then the value of $f'(x)$is 1

(a) $\frac{(f(x))^2}{x(1-f(x)logx)}$ (b) $-\frac{(f(x))^2}{x(1-f(x)logx)}$ (c)$\frac{f(x)}{x(1-f(x)logx)}$ (d)$-\frac{f(x)}{x(1-f(x)logx)}$

9) If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then the value of x is 1

(a) 2 (b) -0.5 (c)1 (d) 0.5

10) If $y = Ae^{5x} + Be^{-5x}$, then $\frac{d^2y}{dx^2}$ is equal to 1

(a) $25y$ (b) $5y$ (c) $-25y$ (d) $15y$

11) The point on the curve $x^2 = 2y$ which is nearest to the point (0, 5) is 1

(a) $(2\sqrt{2}, 4)$ (b) $(2\sqrt{2}, 0)$ (c) $(0,0)$ (d) $(2,2)$

12) A point on the curve $y^2 = 12x$ at which the ordinate increases at twice the rate of abscissa is 1

(a) (2,4) (b) (2, - 4) (c)$(\frac{-9}{8}, \frac{9}{2})$ (d)None of these

13) In a 1000 m race, A reaches the final point in56 seconds and B reaches in 70 seconds. By how much distance does A beats B? 1

(a) 100 m (b) 120 m (c) 150 m (d) 200 m

14) The number of all possible matrices of order 3×3 with each entry 0 or 1 is 1

(a) 18 (b) 27 (c) 81 (d) 512

15) If the demand function is $x = 100 - 4p$,then the quantity at which marginal revenue is equal to zero will be 1

(a) 25 (b) 10 (c) 50 (d) 40

16) If $f(x) = \log x$,then the derivative of $f(\log x)$ with respect to x is 1

(a) $\frac{\log x}{x}$ (b) $\frac{x}{\log x}$ (c) $xlog\ x$ (d) $\frac{1}{xlog\ x}$

17) If A is a square matrix of order 3 x 3 such that $|A| = 4$,then $|3A|$ is equal to 1

(a) 27 (b) 81 (c) 108 (d) 256

18) The function $f(x) = a^x$ is increasing on **R** if 1

(a) $a > 0$ (b) $a < 0$ (c) $0 < a < 1$ (d) $a > 1$

ASSERTION REASON BASED QUESTIONS

In the following questions, a statement of Assertion(A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices

(i) Both A and R are true and R is the correct explanation of A.

(ii) Both A and R are true and R is not the correct explanation of A.

(iii) A is true but R is false.

(iv) A is false but R is true.

19) Assertion(A): If A, B and C are the matrices of order $3 \times l, 2 \times m$ and $3 \times n$ respectively, then $CA + BA$ is defined, if $n = 3$ and $m = 3$. 1

Reason(R): For multiplication of two matrices, A and B i.e. AB is defined if number of columns of A = number of rows of B.

(a) (i) (b) (ii) (c) (iii) (d) (iv)

20) Suppose that $x = t - t^4, y = t^2 - t^3$. 1

Assertion(A):$\frac{dy}{dx}$ at (0,0) is equal to $\frac{2}{3}$ or 1.

Reason(R):$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$.

(a) (i) (b) (ii) (c) (iii) (d) (iv)

SECTION B

(All Questions are compulsory. In case of internal Choice, attempt any one question only)

21) If $y = 3e^{2x} + 2e^{3x}$, then prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$. 2

22) (a) If the matrix $A = \begin{bmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{bmatrix}$ is singular then find the value of x . 2

OR

(b) If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, then find the values of λ so that $A^2 = \lambda A - 2I$. 2

23) Solve the following inequality: $|3x| \geq |6 - 3x|$. 2

24) (a)Find the last three digits of the product 1234×5678. 2

OR

(b)Find the last digit of 12^{12}. 2

25) Find the point on the curve $y = x^2 - 2x + 3$ at which the tangent is parallel to x – axis. 2

SECTION C

(All Questions are compulsory. In case of internal Choice, attempt any one question only)

26) (a) Using properties of determinants, prove that

$$\begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix} = 0 \text{ where } \alpha, \beta, \gamma \text{ are in A.P.}$$ 3

OR

(b) Using properties of determinants, solve the following equation for x: 3

$$\begin{vmatrix} x+a & b & c \\ c & x+b & a \\ a & b & x+c \end{vmatrix} = 0.$$

27) A man wants to cut three lengths from a single piece of cardboard of length 91 cm. The second length is to be 3 cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths for the shortest board if the third piece is to be at least 5 cm longer than the second? 3

28) It is known that the cost of producing 100 units of a commodity is ₹ 250 and cost of producing 200 units is ₹ 300. Assuming that AVC is constant, find the cost function. 3

29) (a) Solve the following system of linear equations using matrix method:

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 9, \frac{2}{x} + \frac{5}{y} + \frac{7}{z} = 52, \frac{2}{x} + \frac{1}{y} - \frac{1}{z} = 0.$ 3

OR

(b) If$A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, then find the value of β such that $A^2 = \beta A - 2I$. Hence find A^{-1}. 3

30) Find the maximum value of $\frac{\log x}{x}, x > 0$. 3

31) Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point (-1,4). 3

SECTION D

(This section comprises of long answer type questions (LA) of 5 marks each)

32) (a) Of all the closed right circular cylinder cans of volume 128π cm^3, find the dimensions of the canwhich has minimum surface area. 5

OR

(b) A company is selling a certain product. The demand function for the product is linear. The company can sell 2000 units when the price is ₹ 8 per unit and it can sell 3000 units when the price is ₹ 4 per unit. Determine the following: 5

(i) demand function (ii) total revenue function

33) Find the remainder when $862 \times 783 \times 671 \times 549 \times 411 \times 395 \times 297$ is divided by 8. 5

34) (a) Using properties of determinants, show that: 5

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

OR

(b) Using properties of determinants show that: 5

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc+bc+ca+ab.$$

35) The sum of three numbers X, Y and Z is 20. If we multiply the first number, X by 2 and add the second number,Y to the result and subtract the third number, Z then we will get the final result as 23. By adding second and third numbers i.e, Y and Z to three times the first number, X then we get 46. Represent the above problem algebraically and use Cramer's rule to find the three numbers X, Y and Z from the equations. 5

SECTION E

(This section comprises of 3 source based questions (Case Studies) of 4 marks each)

36) **Case Study – 1 :**

Two farmers Ravi and Ramu cultivate only three varieties of pulses namely Urad, Massor and Moong. The sale (in ₹) of these varieties of pulses by both the farmers in the month of Septemberand October are given by the following matrices:

$$SEPTEMBER\ SALES = \begin{bmatrix} URAD & MASOOR & MOONG \\ 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix}$$ and

$$OCTOBER\ SALES = \begin{bmatrix} URAD & MASOOR & MOONG \\ 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix}$$.The first row of the matrices indicates the sales by Ravi and the second row of the matrices indicates the sales by Ramu. Based on the above information, answer the following questions.

(a) What are the combined sales of Masoor in September and October for the farmer Ramu? 1

(b) Find the decrease in sales of Moong from September to October for the farmer Ravi. 1

(c) If both the farmers receive 2 % profit on gross sales, then compute the profit for each farmer and for each variety sold in October. 2

OR

(d) Which variety of pulses has the highest selling value in the month of September for the farmer Ramu? 2

37) **Case Study – 2:**

The shape of a toy is given as $f(x) = 6(2x^4 - x^2)$. To make the toy beautiful, 2 sticks which are perpendicular to each other were placed at a point (2, 3), above the toy. Based on the above information,answer the followig questions:

(a) Find the values of abscissa at the critical points. 1

(b) Find the slope of the normal based on the position of the stick. 1

(c) What will be the equation of the tangent at the critical point if it passes through (2, 3)? 2

OR

(d) Find the interval where f(x) is increasing. 2

38)**Case Study – 3:** 4

India faces significant challenges in managing its sewage system due to rapid urbanization and population growth. Efficient management of pipes and cisterns is essential for ensuring proper sewage disposal and preventing environmental pollution.

Let us consider a case study involving two major cities in India, City A and City B, and their sewage systems.

City A: The sewage system in City A consists of a network of pipes and cisterns that collect and transport sewage to treatment plants. The main pipe in City A can drain a tank in 6 hours, while the cistern can fill the same tank in 12 hours. During maintenance, a new pipe is added to the system, which can drain the tank in 4 hours.

City B: The sewage system in City B is undergoing an upgrade to improve efficiency. Initially, it took 8 hours to drain a tank using the existing pipes and cisterns. After the upgrade, a new pipe is added to the system, which can drain the tank in 6 hours.

Compare the efficiency of the sewage systems in City A and City B before and after the upgrades by comparing the rate of draining in both the cities.

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