



# Delhi Public School, Howrah

Periodic Test - II Examination (2024-2025)

Class - XII

Care must be taken not to write anything on the question paper. All the questions must be attempted in the correct sequence.

Subject: - Applied Mathematics (Code No- 241)

Time: 3 Hours

F.M. 80

## General Instructions:

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory.
2. Section - A carries 20 marks weightage, Section - B carries 10 marks weightage, Section - C carries 18 marks weightage, Section - D carries 20 marks weightage and Section - E carries 3 case-based with total weightage of 12 marks.

### Section – A:

3. It comprises of **18 MCQs and 2 Assertion Reason based questions of 1 mark each.**

### Section – B:

4. It comprises of **5 VSA type questions of 2 marks each.**

### Section – C:

5. It comprises of **6 SA type of questions of 3 marks each.**

### Section – D:

6. It comprises of **4 LA type of questions of 5 marks each.**

### Section – E:

7. It has **3 source based/case based/passage based/integrated units of assessment (04 marks each)** with sub parts. Each case study comprises of 3 case-based questions, where **2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks.** Internal choice is provided in **2 marks** question in each case-study.
8. Internal choice is provided in **2 questions in Section - B, 2 questions in Section – C, 2 questions in Section - D.** You have to attempt only one of the alternatives in all such questions.

## SECTION A

(All Questions are compulsory. No internal choice is provided in this section)

- 1) If  $\int x e^{kx^2} dx = \frac{1}{4} e^{2x^2} + C$ , then find the value of k. 1  
(a) 4 (b) 2 (c) 8 (d) None of these
- 2) If  $100 \equiv x \pmod{7}$ , then the least positive value of x is 1  
(a) 2 (b) 3 (c) 1 (d) 4
- 3) Find the slope of the tangent to the curve  $y = x^3 + x$  at the point (1,2). 1  
(a) 2 (b) 4 (c) 3 (d) 0
- 4) The matrix  $A = \begin{bmatrix} 0 & -5 & 3 \\ 5 & 0 & -7 \\ -3 & 7 & 0 \end{bmatrix}$  is a 1  
(a) Diagonal Matrix (b) Symmetric Matrix  
(c) Skew – symmetric Matrix (d) Scalar Matrix
- 5) Find the product of the order and the degree of the following differential equation: 1  
$$1 - \left(\frac{dy}{dx}\right)^2 = \left(a \frac{d^2y}{dx^2}\right)^{\frac{1}{3}}$$
  
(a) 4 (b) 2 (c) 5 (d) 3

- 6) In a 2km race, P can give Q, a start of 200 m and R, a start of 560 m, then in the same race Q can give R a start of 1
- (a) 360 m (b) 380 m (c) 400 m (d) 430 m
- 7) In a Linear Programming Problem, which of the following statement is true? 1
- (a) objective function is linear (b) constraints are linear
- (c) both objective function and constraints are linear (d) None of the above
- 8) If  $\int_0^{40} \frac{dx}{2x+1} = \log k$ , then the value of  $k$  is 1
- (a) 3 (b) 9 (c)  $\frac{9}{2}$  (d) None of these
- 9) The rise in prices before Diwali is an example of 1
- (a) Seasonal Trend (b) Cyclical Trend (c) Long term trend (d) Irregular Trend
- 10) If  $y = 3x - 2$ , then  $\frac{d^2y}{dx^2}$  is equal to 1
- (a) 0 (b)  $6x$  (c) 6 (d) 15
- 11) If A and B are symmetric matrices of same order, then  $AB - BA$  is a 1
- (a) Symmetric Matrix (b) Skew – symmetric Matrix
- (c) Zero matrix (d) Identity Matrix
- 12) If A is a square matrix of order  $3 \times 3$  such that  $|A| = 4$ , then  $|4A|$  is equal to 1
- (a) 27 (b) 81 (c) 108 (d) None of these
- 13) In what ratio shall I add water to the liquid detergent costing ₹ 480 per litre to get resulting mixture worth ₹ 300 per litre? 1
- (a) 5 : 3 (b) 3 : 8 (c) 3 : 5 (d) 5 : 8
- 14) If the points (1,3), (x, 5) and (2,7) are collinear, then the value of x is 1
- (a) 2 (b)  $\frac{3}{2}$  (c) 1 (d)  $\frac{3}{4}$
- 15) If the marginal revenue function of a commodity is  $MR = 2x - 9x^2$ , then the revenue function is 1
- (a)  $2x^2 - 9x^3$  (b)  $2 - 18x$  (c)  $x^2 - 3x^3$  (d)  $18 + x^2 - 3x^3$
- 16) If  $p > q$  and  $r < 0$ , then which of the following is true? 1
- (a)  $pr < qr$  (b)  $p - r < q - r$  (c)  $p + r < q + r$  (d) None of these
- 17) For what value of  $k$ , inverse does not exist for the matrix  $\begin{bmatrix} 1 & 2 \\ k & 6 \end{bmatrix}$ ? 1
- (a) 0 (b) 3 (c) 6 (d) 2
- 18) In a 300 m race, A beats B by 22.5 m or 6 seconds. B's time over the race course is 1
- (a) 80 sec (b) 82 sec (c) 76 sec (d) 90 sec

### ASSERTION REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices

- (i) Both A and R are true and R is the correct explanation of A.  
 (ii) Both A and R are true and R is not the correct explanation of A.  
 (iii) A is true but R is false.  
 (iv) A is false and R is true.

19) Assertion : If  $60 \equiv 4 \pmod{m}$ , then the possible values of  $m$  are 2, 4, 7, 8, 14, 28, 56.

Reason: If  $a \equiv b \pmod{n}$ , then  $a - b$  is an integral multiple of  $n$ , where  $n \in I, n \geq 1$ . 1

- (a) (i) (b) (ii) (c) (iii) (d) (iv)

20) Assertion: If the demand and supply functions for a commodity are  $p_d = x^2 + x - 18$  and  $p_s = 6 + 3x - x^2$ , then the equilibrium price  $p_0 = 2$ .

Reason: At equilibrium, demand = supply. 1

- (a) (i) (b) (ii) (c) (iii) (d) (iv)

## SECTION B

**(All Questions are compulsory. In case of internal Choice, attempt any one question only)**

21) If  $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$  and  $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$ , then find the value of  $x$ . 2

22) (a) If  $x$  is an odd integer, prove that  $x^2 \equiv 1 \pmod{8}$ . 2

**OR**

(b) Find the units digit in  $13^{37}$ . 2

23) Solve for  $x$ :  $-5 \leq \frac{2-3x}{4} \leq 9$  2

24) (a) The supply function for a commodity is  $p = 4 + x$ . Determine producer's surplus if 12 units of goods are sold. 2

**OR**

(b) Solve:  $\frac{dy}{dx} = \frac{y+1}{x-1}$  when  $y(2) = 2$ . 2

25) Evaluate:  $\int_0^1 \frac{e^x dx}{1+e^x}$ . 2

## SECTION C

**(All Questions are compulsory. In case of internal Choice, attempt any one question only)**

26) (a) Using properties of determinants, prove that

$$\begin{vmatrix} a & b-c & c-b \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix} = (a+b-c)(b+c-a)(c+a-b) \quad 3$$

**OR**

(b) Using properties of determinants, solve the following equations for  $x$ : 3

$$\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0.$$

27) Write the integrating factor for the following differential equation: 3

$$x \log x \frac{dy}{dx} + y = 2 \log x$$

28) Solve the following system of linear equations using Cramer's Rule Method:

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \quad \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \quad \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2 \quad 3$$

29) (a) Find two numbers whose sum is 24 and whose product is as large as possible. 3

**OR**

(b) Find the shortest distance of the point  $C(0, c)$  from the parabola  $y = x^2$ . 3

30) From a cask full of alcohol 8 litres are drawn and then filled with water. This process is repeated three more times. The ratio of the quantity of alcohol left in the cask and that of water is 16: 65. How many litres of alcohol did the cask hold originally? 3

31) If  $a, b, c$  are positive real numbers, then find the least value of  $(a + b + c)(ab + bc + ca)$ . 3

## SECTION D

**(This section comprises of long answer type questions (LA) of 5 marks each)**

32) (a) Evaluate:  $\int \frac{x^3}{x^4+3x^2+2} dx$  5

**OR**

(b) Evaluate:  $\int_0^1 \frac{\log x}{\log x + \log(1-x)} dx$  5

33) Find the remainder when  $5^{61}$  is divided by 7. 5

34) (a) Calculate four- Yearly moving averages of number of students studying in a higher secondary school in a particular city from the following data : 5

Year	Number of students
2011	124
2012	120
2013	135
2014	140
2015	145
2016	158
2017	162
2018	170
2019	175

**OR**

(b) Below are given the figures of production (in m. tones) of a rice factory: 5

Year	Production(In m. tones)
2021	80
2013	90
2014	92
2015	83
2016	94
2017	99
2018	92

Fit the straight line trend to the given above figures and calculate trend values.

35) An oil company has two depots A and B with capacities of 7000 litres and 4000 litres respectively. The company is to supply oil for three petrol pumps D, E and F, whose requirements are 4500 litres, 3000 litres and 3500 litres respectively. The distance (in km) between the depots and the petrol pumps is given in the following table:

Distance from the depot(in km)		
From To	A	B
D	7	3
E	6	4
F	3	2

Assuming the transportation cost per km is ₹ 1 per litre, how should the delivery be scheduled in order that the transportation cost is minimum? What is the minimum cost? 5

## SECTION E

**(This section comprises of 3 source based questions (Case Studies) of 4 marks each)**

### 36) Case Study – 1 :

A new restaurant, "Healthy Bites," has opened in town, focusing on nutritious meals. The restaurant offers three main types of dishes: Salads, Soups, and Grain Bowls. The owner, Ms. Priya Malhotra, is keen on ensuring that all meals are balanced in terms of macronutrients—proteins, carbohydrates, and fats. To analyze the nutritional content of each dish type, Ms. Malhotra used the following matrices, where each element represents the amount (in grams) of proteins, carbohydrates, and fats per serving of each dish:

- Matrix A represents the nutritional content of Salads:

$$\begin{bmatrix} 4 & 10 & 5 \\ 3 & 8 & 4 \\ 5 & 9 & 3 \end{bmatrix}$$



Each row in Matrix A corresponds to a different type of Salad (Greek Salad, Caesar Salad, and Garden Salad), and each column represents the amount of protein, carbohydrates, and fats per serving.

• Matrix B represents the nutritional content of Soups:

$$\begin{bmatrix} 3 & 7 & 4 \\ 5 & 6 & 2 \\ 4 & 9 & 3 \end{bmatrix}$$

Each row in Matrix B corresponds to a different type of Soup (Tomato Soup, Lentil Soup, and Vegetable Soup), and each column represents the amount of protein, carbohydrates, and fats per serving.

• Matrix C represents the nutritional content of Grain Bowls:

$$\begin{bmatrix} 6 & 12 & 5 \\ 7 & 15 & 4 \\ 8 & 14 & 6 \end{bmatrix}$$

Each row in Matrix C corresponds to a different type of Grain Bowl (Quinoa Bowl, Brown Rice Bowl, and Barley Bowl), and each column represents the amount of protein, carbohydrates, and fats per serving.

Ms. Malhotra wants to create a balanced meal combo by choosing one Salad, one Soup, and one Grain Bowl. To do this, she needs to analyze the nutritional values of the combos using matrices and determinants. Based on the above information, answer the following questions.

(a) Calculate the total nutritional content (proteins, carbohydrates, and fats) for a meal combo consisting of the first Salad, first Soup, and first Grain Bowl. 1

(b) Find the determinant of Matrix C, representing the nutritional content of the Grain Bowls, to understand the variation in their nutritional values. 1

(c) If Ms. Malhotra wants to scale the nutritional content of the Soups by a factor of 2, what would be the determinant of the scaled matrix  $2B$ ? 2

**OR**

(d) Ms. Malhotra decides to introduce a new menu item by combining all three matrices (Salads, Soups, and Grain Bowls). Calculate the sum of the three matrices  $A + B + C$  and determine the nutritional content of the combo containing the first row of each matrix. 2

### **37) Case Study – 2:**

Sundar Resort is a popular tourist destination located in the foothills of the Himalayas. The management has observed that their revenue depends on the number of visitors they accommodate. To attract more visitors, they occasionally lower the price per room. However, they noticed that while lowering prices can attract more guests, it also decreases the revenue per guest. The relationship between the number of visitors  $n$  and the price per room  $p(n)$  is given by the following equation:

$$p(n) = 5000 - 10n$$

Where:

- $p(n)$  is the price per room in Indian Rupees (₹) when  $n$  guests stay at the resort.
- The resort has 300 rooms available for booking.

The total revenue  $R(n)$  is given by:

$$R(n) = n \times p(n)$$

The resort management wants to maximize their revenue and asks you to determine the optimal number of visitors  $n$  they should aim for.

Based on the above information, answer the following questions:

(a) Express the revenue function  $R(n)$  in terms of  $n$ . 1

(b) Find the derivative of the revenue function  $R(n)$  with respect to  $n$  to determine the rate of change of revenue with respect to the number of guests. 1

(c) Determine the number of visitors  $n$  that maximizes the revenue by setting  $R'(n) = 0$ . 2

**OR**

(d) Calculate the maximum revenue by substituting the optimal number of visitors  $n$  back into the revenue function  $R(n)$ . 2



**38) Case Study – 3:**

Four friends—Ravi, Sita, Gopal, and Meera—embarked on a thrilling river rafting adventure along the Ganga River. Their plan was to row upstream from Point A to a scenic spot at Point B and then return downstream to Point A. The river's current posed a significant challenge to their journey, making it a test of both endurance and strategy. Here are the specifics of their trip:



- ❖ The distance between Point A and Point B is 36 km.
  - ❖ The speed of the raft in still water is 12 km/h.
  - ❖ The speed of the river current is 4 km/h.
  - ❖ The group left Point A at 8:00 AM.
- Based on the above information, answer the following question.

1. Calculate the time taken by the group to reach Point B when rowing upstream. 1
2. After reaching Point B and resting for 30 minutes, they started rowing downstream toward Point A. Calculate the time taken for the return journey to Point A. 1
3. If the group encountered an unexpected increase in the river current by 1 km/h during their downstream journey, what would be the new time taken to return to Point A? 1
4. During their upstream journey, if they decided to take a break after 1.5 hours of rowing, how far from Point A would they be at the time of the break? 1

\*\*\*\*\*