Delhi Public School, Howrah

PERIODIC TEST-II (2024-2025)

Class-XII

Care must be taken not to write anything on the question paper. All the questions must be attempted in the correct sequence.

Subject: - Mathematics (Code No-041)

Time: 3 Hours **F.M. 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

Section- A
(Multiple Choice Questions)
Each question carries 1 mark

1. If a function $f: R \to R$ is defined by

$$f(x) = \begin{cases} 2x, x > 3 \\ x^2, 1 \le x \le 3 \\ 3x, x < 1 \end{cases}$$

Then the value of $f(-1) + f(2) + f(4)$ is

a) 9 b) 14 c)5 d) none of these

2. The domain of the function $\sin 2x + \cos^{-1} 2x$ is

a) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ b) $[-1,1]$ c)R d) $[-1, \pi + 1]$

3. If $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ $and\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then $A^2 - 6A$ is equal to

a) $3I$ b) $-5I$ c) $5I$ d) none of these

4. If A is a square matrix of order 3 such that |A|=9, then the value of |2 adj A| is

a) 648 b) 81 c) 162 d) -162

5. If $f(x) = \begin{cases} mx + 1, x \le \frac{\pi}{2} \\ \sin x + n, x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then which of the following is true?

a) $m = 1, n = 0$ b) $m = \frac{n\pi}{2} + 1$ c) $n = \frac{m\pi}{2}$ d) $m = n = \frac{\pi}{2}$

6. The derivative of $\cos^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right), -\frac{\pi}{4} < x < \frac{\pi}{4}$, w.r.t.x is

a) $\frac{1}{\sqrt{2}}$ b) 1 c)-1 d) none of these

7. If the function $f(x) = x^2 - ax + 5$ is strictly increasing on (1,2), then a lies in the interval

a) $(2, \infty)$ b) $(-\infty, 2)$ c) $(4, \infty)$ d) $(-\infty, 4)$

8. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at

a) $x = 1$ b) $x = 2$ c) $x = -2$ d) $x = -1$

9. If $\int xe^{kx^2}\, dx = \frac{1}{4}e^{2x^2} + C$, then the value of k is

a) 4 b) -2 c) 2 d) 1

10. If $\int_0^k \frac{1}{9x^2+1} dx = \frac{\pi}{12}$, then k is equal to

a) $\frac{\pi}{4}$ b) $\frac{1}{3}$ c) 3 d) none of these

11. The corner points of the feasible region determined by the system of linear inequalities are (0,3),(2,2) and (3,0). If the minimum value of Z=ax+by, a,b>0 occurs at both (0,3) and (2,2), then which of the following is true?

a) a=2b b) 2a=b c) a=b d) 3a=b

12. If the matrix $\begin{bmatrix} 3 & 2x & -5 \\ -4 & 0 & y \\ -5 & 3 & 7 \end{bmatrix}$ is symmetric, then which of the following is true?

a) x=2,y=-3 b) x=2,y=3 c) x=-2,y=-3 d) x=-2,y=3

13. The function $f(x) = x|x|$ at x=0 is

a) continuous but not differentiable
b) differentiable but not continuous
c) continuous and differentiable
d) neither continuous nor differentiable

14. The distance moved by a particle travelling in a straight line in t seconds is given by $s = 45t + 11t^2 - t^3$. Then time taken by the particle to come to rest is

a) 9 sec b) 5/3 sec c)3/5 sec d) 2 sec

15. The least value of the function $f(x) = ax + \frac{b}{x}$, $(x > 0, a > 0, b > 0)$ is

a) $\sqrt{ab}$ b) $2\sqrt{ab}$ c) ab d) $2ab$

16. The optimal value of the objective function of a LPP is attained at the points

a) given by intersection of inequations with the axes only
b) given by intersection of inequations with x-axis only
c) given by corner points of the feasible region
d) none of these

17. $\int_a^b \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a+b-x}} dx$ is equal to

a) $\frac{\pi}{2}$ b)π c)$\frac{1}{2}(b-a)$ d)$b-a$

18. If R is a relation on **R**(set of all real numbers) defined by $xRy \; iff \; x - y + \sqrt{2}$ is an irrational number, then R is

a) reflexive b) symmetric c) transitive d) none of these

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

19. **Assertion(A):** The function $f(x) = x^x, x > 0$ is strictly increasing in $\left(\frac{1}{e}, \infty\right)$

Reason (R): $\log_a x > b \Rightarrow x > a^b \; if \; a > 1$

20. **Assertion(A):** The matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$ is singular.

Reason(R): A square matrix A is said to be singular, if |A|=0.

Section-B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21. Find the value of $\sin^{-1}\left[\cos\left\{\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right\}\right]$

22. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}, show\ that\ A^2 - 5A + 7I = O$. Hence find A^{-1}.

23. (a) Find $\frac{dy}{dx}$, when $x = a\,cos^3\,t, y = a\,sin^3\,t$

OR

(b) If $(\cos x)^y = (\sin y)^x$, find $\frac{dy}{dx}$.

24. Determine the interval in which the function $f(x) = x^4 - \frac{x^3}{3}$ is strictly increasing.

25. (a) If $\int \frac{dx}{1+\sin x} = \tan\left(\frac{x}{2} + a\right) + b$, find the values of a and b.

OR

(b) Evaluate: $\int \frac{e^x \log(\sin e^x)}{\tan e^x} dx$

Section-C

[This section comprises of short answer type questions (SA) of 3 marks each]

26. Solve the following linear programming problem graphically:
Minimize $Z = 3x + 5y$, subject to the constraints $x + 3y \geq 3, x + y \geq 2, x \geq 0, y \geq 0$.

27. (a) If $y = x^n\{a\cos(\log x) + b\sin(\log x)\}, prove\ that\ x^2\frac{d^2y}{dx^2} + (1-2n)\frac{dy}{dx} + (1+n^2)y = 0$.

OR

(b) If $x = a(1+\cos\theta), y = a(\theta + \sin\theta), prove\ that\ \frac{d^2y}{dx^2} = \frac{-1}{a}\ at\ \theta = \frac{\pi}{2}$

28. Show that $f(x) = \sin x(1+\cos x)$ is maximum at $x = \frac{\pi}{3}$ in the interval $[0, \pi]$

29. (a) Evaluate: $\int \frac{2x}{(x^2+1)(x^2+2)^2}\, dx$

OR

(b) Show that $\int_0^{\frac{\pi}{2}} \frac{sin^2x}{\sin x + \cos x} dx = \frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$

30. Using integration, find the area of the region enclosed by the parabola $x^2 = y\ and\ the\ line\ y = x + 2$.

31. Determine the values of a, b and c for which the function

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x} & , for\ x < 0 \\ c & , for\ x = 0 \\ \dfrac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}} & , for\ x > 0 \end{cases}$$

is continuous at x=0.

Section-D

[This section comprises of long answer type questions (LA) of 5 marks each]

32. (a) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, $find\ A^{-1}$. Hence solve the following system of linear equations:

$x + 2y - 3z = -4, 2x + 3y + 2z = 2, 3x - 3y - 4z = 11$

OR

(b) If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ $and\ B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are two square matrices, find AB and hence solve the system of linear equations: $x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7$.

33. (a) Let $A = \{x \in Z: 0 \leq x \leq 12\}$. Show that the relation $R = \{(a,b): a,b \in A, |a-b| is\ divisible\ by\ 4\}$ is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2].

OR

(b) Let $A = R - \{3\}, B = R - \{1\}. Let\ f: A \to B$ be defined by $f(x) = \frac{x-2}{x-3}$, $\forall\ x \in A$. Show that f is bijective.

34. Show that of all the rectangles inscribed in a given circle, the square has the maximum area.

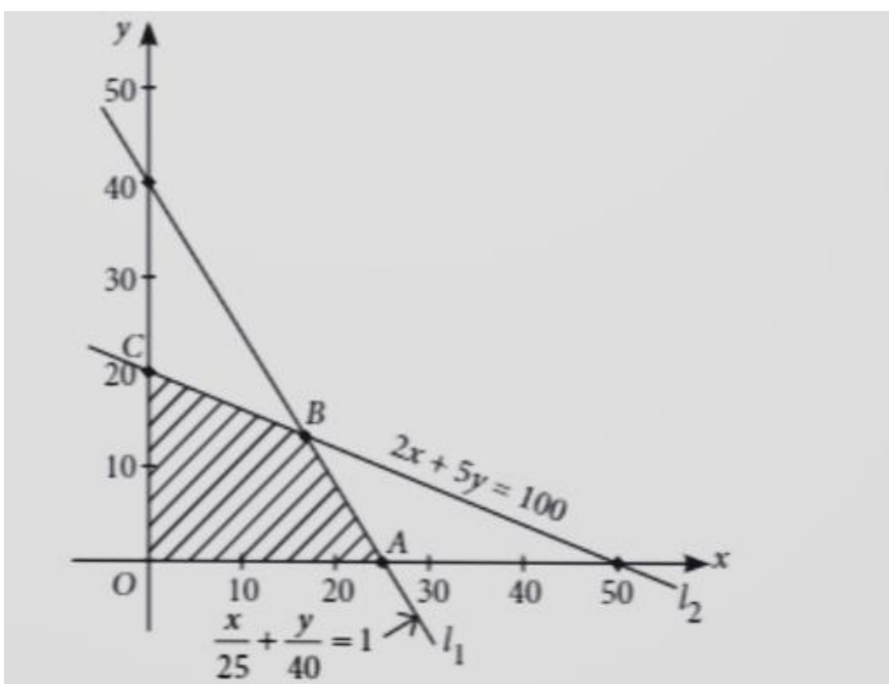
35. If $f(x) = \begin{cases} x^2 + 3x + a, for\ x \leq 1 \\ bx + 2, for\ x > 1 \end{cases}$ is everywhere differentiable, find the values of a and b.

Section-E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.]

36. **Case Study-1:**

The feasible region of an LPP is shown below.



Based on the information given above, answer the following questions:

(i) Find the point of intersection of the lines l_1 and l_2. 1

(ii) Find the corner points of the feasible region. 1

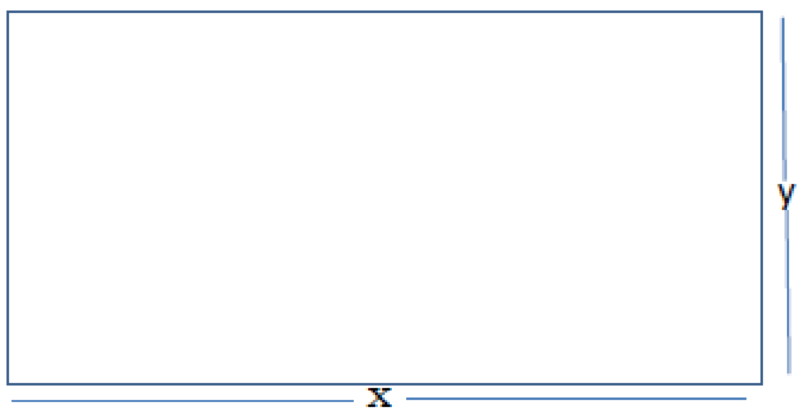
(iii) (a) Determine the linear constraints of the above LPP. 2

OR

(b) If Z=x+y be the objective function and $Z_{max} = 30$, then find the corner point of the feasible region where the maximum value of Z occurs. 2

37. **Case Study-2:**

Manjit wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50m, then its area will remain same, but if length is decreased by 10m and breadth is decreased by 20m, then its area will decrease by 5300 m^2. Consider the length and breadth of the plot be x m and y m respectively as shown in the figure given below.



Based on the information given above, answer the following questions:

(i) Frame the equations in terms of x and y which represent the above situation. 1

(ii) Write the matrix form of the equations obtained in question (i). 1

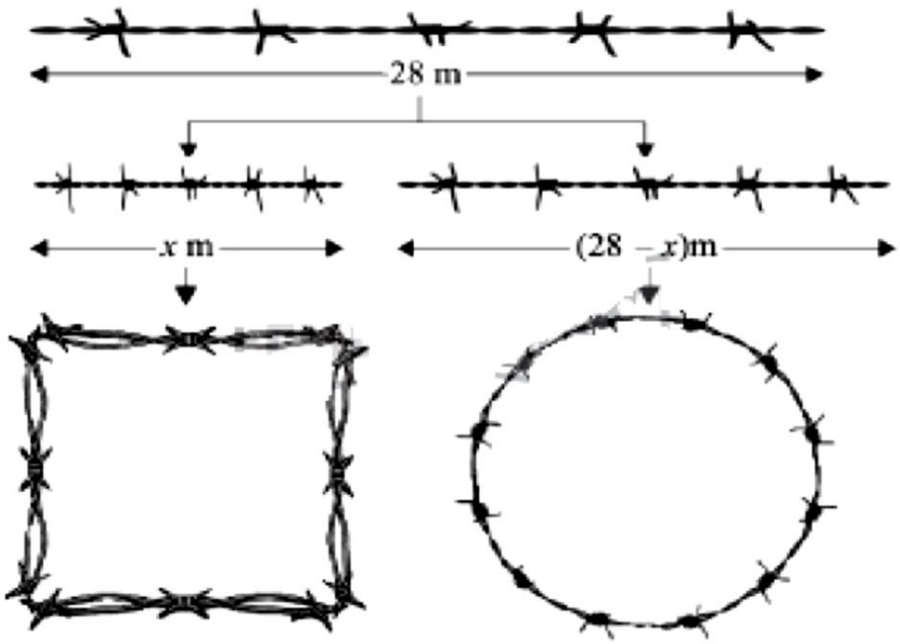
(iii) (a) Find the length of the rectangular plot. 2

OR

(b) Find the breadth of the rectangular plot. 2

38. **Case Study-3:**

Rahul bought a wire of length 28 m. The wire is to be cut into two pieces; one of the pieces is to made into a square and the other into a circle as shown in the figure below:



Based on the information given above, answer the following questions:

(i) Write the combined area of circle and square in terms of x. 2

(ii) Find the minimum combined area of circle and square. 2