Delhi Public School, Howrah

PRE-BOARD EXAMINATION (2024-2025)

Class-XII

Care must be taken not to write anything on the question paper. All the questions must be attempted in the correct sequence.

Subject: - Mathematics (Code No-041)

Time: 3 Hours **F.M. 80**

General Instructions:

(i) This Question paper contains 38 questions. All questions are compulsory.
(ii) This Question paper is divided into five Sections - A, B, C, D and E.
(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
(ix) Use of calculators is not allowed.

Section- A

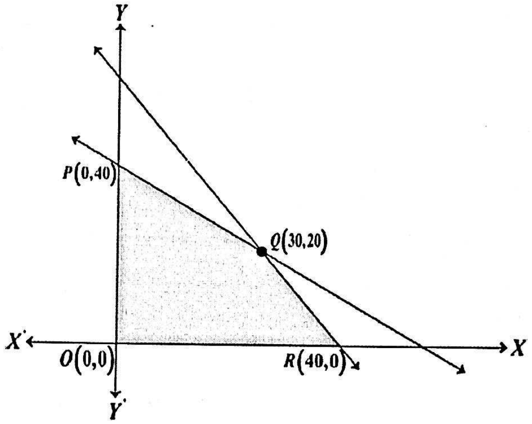
(This section comprises of multiple choice questions (MCQs) of 1 mark each)

1. If for a square matrix A, $A.(adj\ A) = \begin{bmatrix} 2025 & 0 & 0 \\ 0 & 2025 & 0 \\ 0 & 0 & 2025 \end{bmatrix}$, then the value of |A|+|adj A| is equal to
   a) 1 b) 2025+1 c) $(2025)^2 + 45$ d) $2025 + (2025)^2$
2. If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ $and\ B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then B is equal to
   a) $I\cos\theta + J\sin\theta$ b) $I\sin\theta + J\cos\theta$ c) $I\cos\theta - J\sin\theta$ d) $-I\cos\theta + J\sin\theta$
3. The function $f(x) = x^x$ decreases in the interval
   a) (0,e) b) (0,1) c) (0,1/e) d) (1/e,e)
4. If B is a non-singular matrix and A is a square matrix, then det($B^{-1}AB$) is equal to
   a) det(A^{-1}) b) det(B^{-1}) c) det(A) d) det(B)
5. The degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$ is
   a) 1 b) 2 c) 3 d) not defined
6. The area of a triangle with vertices (-3,0),(3,0) and (0,k) is 9 sq. units. The value of k will be
   a) 9 b) 3 c) -9 d) 6
7. If A and B are symmetric matrices of same order, then AB-BA is a
   a) symmetric matrix b) skew-symmetric matrix c) zero matrix d) identity matrix
8. If $P(A) = \frac{4}{5}\ and\ P(A \cap B) = \frac{7}{10}, then\ P(B|A)$ is equal to
   a) $\frac{1}{10}$ b) $\frac{1}{8}$ c) $\frac{7}{8}$ d) $\frac{17}{20}$
9. If $|\vec{a}| = 3\ and\ |\vec{b}| = 4$, then the value of λ for which $\vec{a} + \lambda\vec{b}\ and\ \vec{a} - \lambda\vec{b}$ are perpendicular is
   a) $\frac{9}{16}$ b) $\frac{3}{4}$ c) $\frac{3}{2}$ d) $\frac{4}{3}$

10. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$,then the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ is

a)1 b)$\frac{3}{2}$ c) $-\frac{3}{2}$ d)none of these

11. For the linear programming problem (**LPP**), the objective function is Z=4x+3y and the feasible region determined by a set of constraints is shown in the graph:



Which of the following statements is true?

a) Maximum value of *Z* is at R(40,0) .

b) Maximum value of *Z* is at Q(30,20) .

c) The value of *Z* at R(40,0) is less than the value at P(0,40).

d) The value of *Z* at Q(30,20) is less than the value at R(40,0) .

12. $\int \frac{dx}{x^2(x^4+1)^{\frac{3}{4}}}$ equals

a)$-\frac{1}{x}(x^4+1)^{\frac{1}{4}} + C$ b) $\frac{1}{x}(x^4+1)^{\frac{1}{4}} + C$ c) $-\frac{1}{x^2}(x^4+1)^{\frac{1}{4}} + C$ d) $\frac{1}{x^2}(x^4+1)^{\frac{1}{4}} + C$

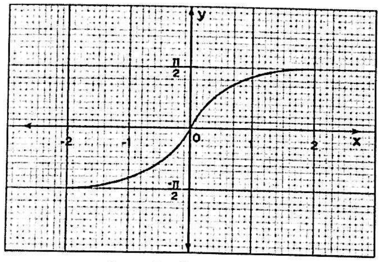
13. The value of $\int_0^1 \frac{\tan^{-1}x}{1+x^2}\, dx$ is

a)$\frac{\pi}{4}$ b)$\frac{\pi^2}{32}$ c)1 d) $\frac{\pi^2}{16}$

14. The general solution of the differential equation $\frac{dy}{dx} + y\tan x = \sec x$ is

a)$y\sec x = \tan x + C$ b) $y\tan x = \sec x + C$ c) $x\sec x = \tan y + C$ d) None of these

15. The graph drawn below depicts



a) $y = \sin^{-1} x$ b) $y = \cos^{-1} x$ c) $y = \tan^{-1} x$ d) $y = \cot^{-1} x$

16. The corner points of the feasible region for an LPP are (0,10), (5,5), (15,15) and (0,20). If the objective function is Z=px+qy, p,q>0, then the condition on p and q so that the maximum of Z occurs at (15,15) and (0,20) is

a)p=q b)p=2q c)q=2p d)q=3p

17. The function f(x)=x-[x], where [.] denotes the greatest integer function is

a)continuous everywhere b) continuous at integer points only

c) continuous at non-integer points only d) differentiable everywhere

18. The area enclosed between the curve $y=4x-x^2$ and the x-axis is

a) $\frac{3}{32}$ sq. units b) $\frac{32}{3}$ sq. units c) $\frac{64}{3}$ sq. units d) $\frac{3}{64}$ sq. units

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

19. Consider the function $f(x) = \begin{cases} x^2, x \geq 1 \\ x+1, x < 1 \end{cases}$

Assertion(A): f is not differentiable at x=1 as $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$

Reason (R): If a function f is differentiable at a point 'a', then it is continuous at 'a'.

20. **Assertion(A):** If R is a relation defined on the set of natural numbers N such that $R = \{(x,y): x,y \in N \text{ and } 2x + y = 24\}$, then R is an equivalence relation.

Reason (R): A relation is said to be an equivalence relation if it is reflexive, symmetric, and transitive.

Section-B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21. Find the principal value of $\cot[\sin^{-1}\{\cos(\tan^{-1} 1)\}]$

22. The total cost $C(x)$ associated with the production of x units of an item is given by

$$C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$$

Find the marginal cost when 3 units are produced.

23. (a) Find the derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\tan^{-1} x, -1 < x < 1$.

OR

(b) If $x^m y^n = (x+y)^{m+n}, \text{prove that } \frac{dy}{dx} = \frac{y}{x}$.

24. (a) If $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$, then find a unit vector which is perpendicular to both the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$.

OR

(b) The vectors $\vec{a} = 3\hat{i} + x\hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} + y\hat{k}$ are mutually perpendicular. Given that $|\vec{a}| = |\vec{b}|$, find the values of x and y.

25. Prove that the points with position vectors $\hat{i} - \hat{j}$, $4\hat{i} - 3\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + 5\hat{k}$ are the vertices of a right-angled triangle.

Section-C

[This section comprises of short answer type questions (SA) of 3 marks each]

26. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2 m/sec. How fast its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

27. A beam of length l is supported at two ends and is uniformly loaded. If W is the uniform load per unit length, the bending moment M at a distance x from one end is given by $M = \frac{W}{2}lx - \frac{1}{2}Wx^2$. Find the point on the beam at which the bending moment has the maximum value.

28. (a) An ant is moving along the vector $\vec{l_1} = \hat{i} - 2\hat{j} + 3\hat{k}$. Few sugar crystals are kept along the vector $\vec{l_2} = 3\hat{i} - 2\hat{j} + \hat{k}$ which is inclined at an angle θ with the vector $\vec{l_1}$. Find the angle θ. Also find the scalar projection of $\vec{l_1}$ on $\vec{l_2}$.

OR

(b) Find the value of p, so that the lines $l_1: \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$ and $l_2: \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other. Also find the equation of a line passing through the point (3,2,-4) and parallel to l_1.

29. (a) Evaluate: $\int e^x(\tan x - \log \cos x)\,dx$

OR

(b) Evaluate: $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}}\,dx$

30. Solve the following Linear Programming Problem graphically:

$Maximize\ Z = 5x + 2y, subject\ to\ x - 2y \le 2, 3x + 2y \le 12, -3x + 2y \le 3, x \ge 0, y \ge 0.$

31. (a) The probability distribution of a random variable X is given below:

X	0	1	2	3
P(X)	k	k/2	k/4	k/8

(i) Determine the value of k.

(ii) Determine $P(X \le 2) and\ P(X > 2)$.

OR

(b) From a lot of 10 bulbs, which includes 3 defective bulbs, a sample of 2 bulbs is drawn at random. Find the probability distribution of defective bulbs.

Section-D

[This section comprises of long answer type questions (LA) of 5 marks each]

32. Draw a rough sketch of the region $\{(x,y): x^2 + y^2 \le 4, x + y \ge 2\}$ and use integration to find its area.

33. (a) Find the shortest distance between the lines $\vec{r} = 3\hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ and $\vec{r} = 5\hat{\imath} - 2\hat{\jmath} + \mu(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$, where λ and μ are parameters.

OR

(b) A line passing through the point A with position vector $\vec{a} = 4\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ is parallel to the vector $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$. Find the length of perpendicular drawn on this line from a point P with position vector $\vec{p} = 4\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$.

34. (a) An owl was sitting at (0,k);k>0 . Then it starts flying along the path whose equation is given by $y = ax^2 + bx + c, where\ a \in R - \{0\}, b, c \in R$. It passes through the points (1,2), (2,1), (4,5). Determine the values of a, b and c by solving the system of linear equations in a, b and c, using matrix method and hence find k.

OR

(b) A toy rocket is fired, from a platform, vertically into the air, its height above the ground after t seconds is given by $s(t) = at^2 + bt + c, where\ a, b, c \in R;\ a \neq 0\ and\ s(t)$ is measured in metres. After 10 second, the rocket is 16 m above the ground; after 20 seconds, 22 m; after 30 seconds, 25 m.

(i) Write down a system of three linear equations in terms of a, b and c.

(ii) Hence find the values of a,b and c using matrix method.

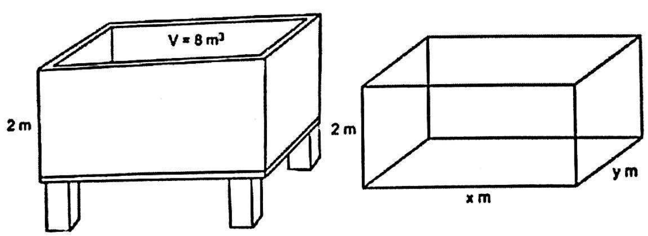
35. If $x = a\cos\theta + b\sin\theta\ and\ y = a\sin\theta - b\cos\theta$, prove that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$.

Section-E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.]

36. **Case Study-1:**

On the request of villagers, a construction agency designs a tank with the help of an architect. The tank consists of a rectangular base with rectangular sides, open at the top so that its depth is 2 m and volume is 8 m^3 as shown below. The construction of the tank costs Rs 70 per sq. m for the base and Rs 45 per sq. m for sides.



Based on the information given above, answer the following questions:

(i) If x and y represent the length and breadth of its rectangular base, then find the relation between them. 1

(ii) Express making cost C in terms of length of rectangular base. 1

(iii) (a) Find the value of x so that the cost of construction is minimum. 2

OR

(b) Verify by second derivative test that cost is minimum at a critical point. 2

37. **Case Study-2:**

In two different societies, there are some school going students including girls as well as boys. Satish forms two sets with these students for his college project. Let A= {a_1, a_2, a_3, a_4, a_5 } and B= {b_1, b_2, b_3, b_4 } where a_i's and b_i's are school going students of first and second societies respectively. Satish decides to explore these sets for various types of relations and functions.

Based on the above information, answer the following questions:

(i) Satish wishes to know the number of reflexive relations defined on set A. How many such relations are possible? 1

(ii) Satish defines a relation R on A such that $R = \{(x, y): x \text{ and } y \text{ are students of same sex}\}$. Is the relation R transitive? Justify your answer. 1

(iii) (a) Satish and his friend Rajat are interested to know the number of symmetric relations defined on both the sets A and B, separately. Satish decides to find the symmetric relation on set A, while Rajat decides to find the symmetric relation on set B. What is the difference between their results? 2

OR

(b) To help Satish in his project, Rajat decides to form onto functions from set A to B. How many such functions are possible? 2

38. **Case Study-3:**

Arka bought two cages of birds: Cage-I contains 5 parrots and 1 owl and Cage –II contains 6 parrots. One day Arka forgot to lock both cages and two birds flew from Cage-I to Cage-II (simultaneously).Then two birds flew back from cage-II to cage-I(simultaneously). Assume that all the birds have equal chances of flying.

On the basis of the above information, answer the following questions:-

(i) When two birds flew from Cage-I to Cage-II and two birds flew back from Cage-II to Cage-I then find the probability that the owl is still in Cage-I. 2

(ii) When two birds flew from Cage-I to Cage-II and two birds flew back from Cage-II to Cage-I, the owl is still seen in Cage-I, what is the probability that one parrot and the owl flew from Cage-I to Cage-II? 2