Delhi Public School, Howrah

PERIODIC TEST-III (2024-2025)

Class-XII

Care must be taken not to write anything on the question paper. All the questions must be attempted in the correct sequence.

Subject: - Mathematics (Code No-041)

Time: 3 Hours **F.M. 80**

General Instructions:

(i) This Question paper contains 38 questions. All questions are compulsory.
(ii) This Question paper is divided into five Sections - A, B, C, D and E.
(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
(ix) Use of calculators is not allowed.

Section- A

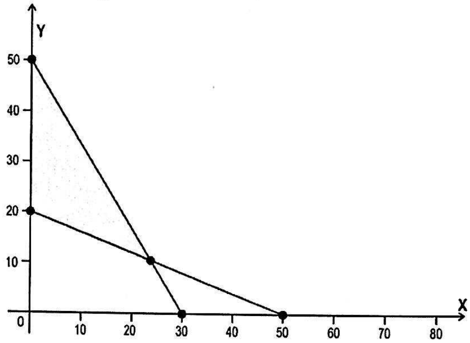
(This section comprises of multiple choice questions (MCQs) of 1 mark each)

1. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then the value of $|adj\ A|$ is
   a) a^{27} b) a^9 c) a^6 d) a^2
2. If A is a matrix of order $m \times n$ and B is a matrix such that AB^T *and* B^TA are both defined, then the order of matrix B is
   a) $m \times n$ b) $n \times n$ c) $n \times m$ d) $m \times m$
3. If the function $f(x) = 2x^2 - kx + 5$ is increasing on [1,2], then k lies in the interval
   a) $(-\infty, 4)$ b) $(4, \infty)$ c) $(-\infty, 8)$ d) $(8, \infty)$
4. If A and B are non-singular matrices of same order with det(A)=5, then $\det(B^{-1}AB)^2$ is equal to
   a) 5 b) 5^2 c) 5^4 d) 5^5
5. The number of arbitrary constants in the particular solution of a differential equation of second order is(are)
   a) 3 b)2 c)1 d) 0
6. If $\begin{vmatrix} 3x & 4 \\ 5 & x \end{vmatrix} = \begin{vmatrix} 4 & -3 \\ 5 & -2 \end{vmatrix}$, then x=?
   a)3 only b)-3 only c)3 or -3 d)6 or -6
7. If A,B are two non-singular matrices of same order, then which of the following is true?
   a)AB is non-singular b)AB is singular c)$(AB)^{-1} = A^{-1}B^{-1}$ d)AB is not invertible
8. If A and B are two independent events with $P(A) = \frac{3}{5}$ *and* $P(B) = \frac{4}{9}$, *then* $P(A' \cap B')$ is equal to
   a)4/15 b)8/45 c)1/3 d)2/9
9. The angle between two vectors $\vec{a}$ *and* $\vec{b}$ with magnitudes $\sqrt{3}$ *and* 4, respectively and $\vec{a}.\vec{b} = 2\sqrt{3}$ is
   a)$\frac{2\pi}{3}$ b)$\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{6}$

10. The value of α if the angle between the vectors $\vec{p} = 2\alpha^2\hat{\imath} - 3\alpha\hat{\jmath} + \hat{k}$ and $\vec{q} = \hat{\imath} + \hat{\jmath} + \alpha\hat{k}$ is obtuse, is

a)R-[0,1] b)(0,1) c) $[0, \infty)$ d) $[1, \infty)$

11. Opticare Pvt. Ltd. is constructing an analysis of its operational costs, where labour cost is represented as x and the raw material is represented as y. The cost optimization is framed as a linear programming problem (LPP).
Observe the graph of the feasible region of the cost optimization LPP shaded below.



Which of the following inequalities is one of the constraints of the LPP?

a) $5x + 3y \geq 150$ b) $5y + 3x \leq 150$ c) $2x + 5y \geq 100$ d) $2y + 5x \leq 100$

12. If $f'(x) = 4x^3 - \frac{3}{x^4}$, then f(x) is equal to

a)$x^4 + \frac{1}{x^3} + C$ b) $x^4 - \frac{1}{x^3} + C$ c) $x^3 + \frac{1}{x^3} + C$ d)none of these

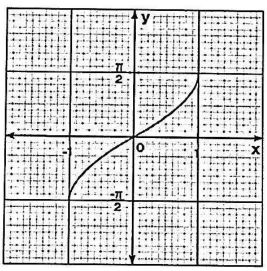
13. The value of $\int_{-1}^{1} \sin^5 x \cos^4 x \, dx$ is equal to

a)$\frac{1}{2}$ b)1 c)0 d)$\frac{\pi^2}{4}$

14. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ is

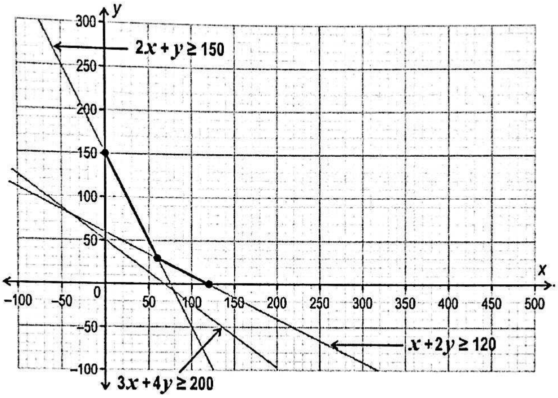
a)$\phi\left(\frac{y}{x}\right) = kx$ b) $x\phi\left(\frac{y}{x}\right) = k$ c) $\phi\left(\frac{y}{x}\right) = ky$ d) $y\phi\left(\frac{y}{x}\right) = k$

15. The graph drawn below depicts



a) $y = \sin^{-1} x$ b) $y = \cos^{-1} x$ c) $y = \tan^{-1} x$ d) $y = \cot^{-1} x$

16. The objective function of a linear programming problem (LPP), Z=4x+3y, has to be minimized. The feasible region of this LPP, along with its constraints, is shown in the graph below.



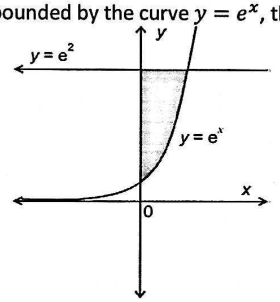
Which constraint, if removed, will not affect the feasible region?

a) $x + 2y \geq 120$ b) $2x + y \geq 150$ c) $3x + 4y \geq 200$ d) any of the given constraints, if removed, will affect the feasible region

17. The value of k for which the function $f(x) = \begin{cases} \frac{kx}{|x|}, if\ x < 0 \\ 3, if\ x \geq 0 \end{cases}$ is continuous at x=0 is

a) 3 b) -3 c) 0 d) 1

18. The shaded region shown below is bounded by the curve $y = e^x$, the y-axis and the line $y = e^2$.



Which of the following is the area of the shaded region?

a) $\int_1^{e^2} (e^2 - e^x)\, dx$ b) $\int_1^{2} (e^2 - e^x)\, dx$ c) $\int_1^{e^2} e^x\, dx$ d) $\int_0^{2} e^x\, dx$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

19. Consider the function $f(x) = [\sin x], x \in [0, \pi]$.

Assertion(A): f(x) is continuous at $x = \frac{\pi}{2}$.

Reason (R): $\lim_{x \to \frac{\pi}{2}} f(x)$ does not exist.

20. **Assertion(A):** Let A and B be sets. Then the function $f: A \times B \to B \times A$ *such that* $f(a,b) = (b,a)$ is bijective.

Reason (R): A function f is said to be bijective, if it is both one-one and onto.

Section-B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21. Find the value of $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

22. The total cost $C(x)$ associated with the production of x units of an item is given by

$$C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$$

Find the marginal cost when 17 units are produced.

23. (a) Find the derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1} x, x \neq 0$.

OR

(b) Differentiate the following function with respect to x: $(\sin x)^{\log x}$

24. (a) If $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} + \hat{\jmath} - 2\hat{k}, \vec{c} = \hat{\imath} + 3\hat{\jmath} - \hat{k}$, *find* λ *such that* $\vec{a}$ *is perpendicular to* $\lambda\vec{b} + \vec{c}$.

OR

(b) The projection of a vector $\vec{a} = 3\hat{\imath} + q\hat{\jmath} - \hat{k}$ on vector $\vec{b} = \hat{\imath} + \sqrt{2}\hat{\jmath} + p\hat{k}$ is 1, where p, q are natural numbers. If $|\vec{b}| = \sqrt{12}$, find p *and* q.

25. A parallelogram ABCD is constructed such that its adjacent sides AB and AD are $3\vec{a} - 5\vec{b}$ and $\vec{a} - 2\vec{b}$ respectively. $|\vec{a}| = \sqrt{2}, |\vec{b}| = \sqrt{8}$ and the angle between $\vec{a}$ *and* $\vec{b}$ is $\frac{\pi}{3}$. Find the length of the diagonal BD.

Section-C

[This section comprises of short answer type questions (SA) of 3 marks each]

26. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

27. A cylindrical disk of radius R and height H is pressed by a hydraulic press. During the process, the radius and the height of the disk change in such a way that its shape is retained and volume remains constant. Find the ratio of the rate of change of height to the rate of change of radius in terms of R.

28. (a) If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices A,B,C of a triangle ABC, show that the area of the triangle ABC is $\frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$. Hence deduce the condition for the points $\vec{a}, \vec{b}, \vec{c}$ to be collinear.

OR

(b) Find the vector equation of a line passing through a point with position vector $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and parallel to the line joining the points $-\hat{\imath} + 4\hat{\jmath} + \hat{k}$ *and* $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$. Also find the cartesian equation of the line.

29. (a) Evaluate: $\int \frac{(x-1)}{(x+1)(x-2)} dx$

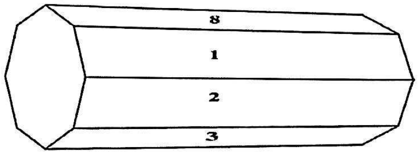
OR

(b) Evaluate: $\int_0^{2\pi} e^x \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) dx$

30. Solve the following Linear Programming Problem graphically:

$Maximize\ Z = 6x + 3y, subject\ to\ 4x + y \geq 80, x + 5y \geq 115, 3x + 2y \leq 150, x \geq 0, y \geq 0.$

31. (a) An octagonal prism is a three-dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces. It has 24 edges and 16 vertices.



The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted. Let X denotes the number obtained on the bottom face and the following table gives the probability distribution of X.

X	1	2	3	4	5	6	7	8
P(X)	p	2p	2p	p	2p	P^2	$2p^2$	$7p^2$+p

On the above context, answer the following questions.

(i) Find the value of p.

(ii) Find the mean, E(X).

OR

(b) There are two boxes I and II. Box I contains 3 red and 6 black balls. Box II contains 5 red and n black balls. One of the two boxes, box I and box II is selected at random and a ball is drawn from it. The ball drawn is found to be red. If the probability that this red ball comes out from box II is $\frac{3}{5}$, find the value of n.

Section-D

[This section comprises of long answer type questions (LA) of 5 marks each]

32. Draw a rough sketch of the curve $y = x^2 - 5x + 6$. Using integration find the area bounded by the curve and the x-axis.

33. (a) Find the shortest distance between the following pair of lines and determine whether they intersect or not:

$$\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5} \text{ and } \frac{x-8}{7} = \frac{y-7}{1} = \frac{z-5}{3}$$

OR

(b) Find the image of the point (1,2,1) with respect to the line $\frac{x-3}{1} = \frac{y+1}{2} = \frac{z-1}{3}$. Also find the equation of the line joining the given point and its image.

34. An insurance company agent has the following record of policies sold in the month of April, May and June 2022 for three different policies-Policy A, Policy B and Policy C. He is paid a fixed commission per policy sold but the commission varies for the policies A,B and C.

Months	Number of policies Sold			Total commission earned in the month (in Rs)
	Policy A	Policy B	Policy C	
April	8	4	6	7850
May	9	9	6	9600
June	12	9	12	15000

Find the fixed commission payable on policies A,B and C per unit.

35. (a) If $x = a(\theta - \sin\theta), y = a(1 - \cos\theta), find\ \frac{d^2y}{dx^2}\ at\ \theta = \frac{\pi}{3}$.

OR

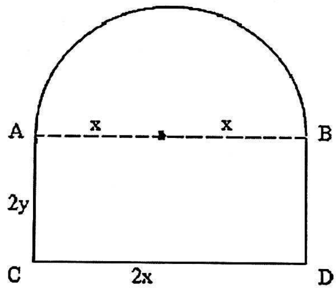
(b) Show that the function $f(x) = |x - 1| + |x + 1|, for\ all\ x \in R$ is not differentiable at the points $x = -1\ and\ x = 1$.

Section-E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.]

36. **Case Study-1:**

Mr Shashi, who is an architect, designs a building for a small company. The design of window on the ground floor is proposed to be different than other floors. The window is in the shape of a rectangle which is surmounted by a semi-circular opening. This window is having a perimeter of 10 m as shown below:



Based on the information given above, answer the following questions:

(i) If 2x and 2y represent the length and breadth of the rectangular portion of the window, then find the relation between x and y. 1

(ii) Find the combined area (A) of the rectangular region and semi-circular region of the window as a function of x. 1

(iii) (a) The owner of this small company is interested in maximizing the area of the whole window so that maximum light input is possible. Find the length of rectangular portion of the window so that the area of the whole window becomes maximum. 2

OR

(b) Find the maximum area of the whole window. 2

37. **Case Study-2:**

An inspection was conducted in a Govt. School at Delhi. The inspection team visited class XII and selected two sets A,B of three students each.

$A = \{b_1, b_2, b_3\}$ *and* $B = \{g_1, g_2, g_3\}$, where b_i, g_i represents particular boy, girl respectively,$i = 1,2,3$.

Based on the above information, answer the following questions:

(i) How many relations can be defined from A to B? 1

(ii) How many functions can be defined from A to B? 1

(iii) (a) Find the number of reflexive relations on set A. 2

OR

(b) Find the number of equivalence relations on set A. 2

38. **Case Study-3:**

A hospital conducts a screening test for a rare disease. It is known that:

a) The probability of a person having the disease is **0.01** (i.e., 1% of the population has the disease).

b) The test is **99% accurate**, meaning:

- If a person has the disease, the test will correctly identify it **99% of the time** (true positive).
- If a person does not have the disease, the test will correctly report a negative result **99% of the time** (true negative).

However, the test is not perfect and may give false positives and false negatives:

- **False Positive:** The test incorrectly identifies the disease in a healthy person 1% of the time.
- **False Negative:** The test fails to identify the disease in an affected person 1% of the time.

A person is selected randomly from the population and tests **positive** for the disease.

Based on the above information, answer the following questions:

(i) What is the probability that the person selected actually has the disease, given that they tested positive? 2

(ii) What is the probability that a person does not have the disease given that they tested positive? 2