07.07.25

Delhi Public School, Howrah

PERIODIC TEST-I (2025-2026)

Class-XII

Care must be taken not to write anything on the question paper. All the questions must be attempted in the correct sequence.

Subject: - Mathematics (Code No-041)

Time: 3 Hours **F.M. 80**

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This question paper contains 38 questions. All questions are compulsory.

(ii) This question paper is divided into five Sections – A, B, C, D and E.

(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.

(iv) In Section B, Questions no. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.

(v) In Section C, Questions no. 26 to 31 are short answer (SA) type questions, carrying 3 marks each.

(vi) In Section D, Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.

(vii) In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

(ix) Use of calculators is not allowed.

Section- A
(Multiple Choice Questions)
Each question carries 1 mark

1. The range of the principal value branch of $\sec^{-1} x$ is

   A) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ B) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ C) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ D) $(0, \pi)$

2. If $f(x) = \begin{cases} \frac{\sqrt{x^2+7}-4}{x+3}, & x \neq -3 \\ k, & x = -3 \end{cases}$ is continuous at x=-3, then the value of k is

   A) $\frac{3}{4}$ B) 0 C) $-\frac{3}{4}$ D) None of these

3. If A is a square matrix of order 3 such that $A(adj\ A) = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$, then |A| is equal to

   A) 9 B) -3 C) -6 D) 3

4. If $A = \begin{bmatrix} 0 & a & 1 \\ -1 & b & 1 \\ -1 & c & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the value of $(a+b+c)^2$ is

   A) 1 B) 0 C) 4 D) none of these

5. If A and B are invertible matrices of same order, then which of the following statement is not true?

   A) $|A^{-1}| = |A|^{-1}$ B) $adj\ A = |A|A^{-1}$ C) $(A+B)^{-1} = B^{-1} + A^{-1}$ D) $(AB)^{-1} = B^{-1}A^{-1}$

6. A relation R in the set A = {1,2,3} is defined as R = {(1, 1), (1, 2), (2, 2), (3, 3)}. Which of the following ordered pair in R shall be removed to make it an equivalence relation in A?

   A) (1,1) B) (1,2) C) (2,2) D) (3,3)

7. If A,B, C are non-singular matrices of same order, then $(AB^{-1}C)^{-1} =$?

   A) CBA^{-1} B) $C^{-1}B^{-1}A^{-1}$ C) $C^{-1}BA^{-1}$ D) $C^{-1}BA$

8. What is the domain of the function $y = \sec^{-1} x + \sin^{-1} x$?

   A) -1 and 1 B) $[-1,1]$ C) $(-\infty, 1] \cup [1, \infty)$ D) $\emptyset$

9. Let T be the set of all triangles in the Euclidean plane, and let a relation R on T be defined as aRb if a is congruent to b for all a,b∈ T. Then R is
   A)reflexive but not symmetric
   B)transitive but not symmetric
   C)equivalence
   D)none of these

10. If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ $and\ kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of k, a and b respectively are:
   A) -6, -12, -18 B) -6, -4, -9 C) -6, 4, 9 D)-6, 12, 18

11. If $x = \sqrt{a^{\sin^{-1} t}}, y = \sqrt{a^{\cos^{-1} t}}$ $(a > 0\ and\ t \in (-1,1)), then$ $\frac{dy}{dx}$ is equal to
   $A)$ $-\frac{y}{x}$ B) $\frac{y}{x}$ C) $\frac{x}{y}$ D) $-\frac{x}{y}$

12. The number of points of discontinuity of the rational function $f(x) = \frac{x^2-3x+2}{4x-x^3}$ is
   A)1 B) 2 C) 3 D) 4

13. The matrix $\begin{bmatrix} 2 & -1 & 3 \\ \lambda & 0 & 7 \\ -1 & 1 & 4 \end{bmatrix}$ is not invertible for which value of λ?
   A)-1 B) 1 C) 0 D) $R - \{1\}$

14. If A and B are non-singular matrices of same order, then choose the correct option:
   A)AB is non-singular always
   B)AB is singular always
   $C)(AB)^{-1} = A^{-1}B^{-1}$
   D)AB is not invertible

15. Let $f(x) = \begin{cases} ax^2 + 1, x > 1 \\ x + \frac{1}{2} \quad, x \le 1 \end{cases}$ .Then f(x) is derivable at x=1, if
   A)a=2 B) a=1 C) a=0 D) a=1/2

16. The function $f: [0, \infty) \to R\ given\ by\ f(x) = \frac{x}{x+1}$ is
   A)one-one and onto
   B)one-one but not onto
   C)onto but not one-one
   D)neither one-one nor onto

17. If $f(x) = \tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}, 0 \le x \le \frac{\pi}{2}$, then $f'\left(\frac{\pi}{6}\right)$ is
   $A)$ $-\frac{1}{4}$ B)$-\frac{1}{2}$ C) $\frac{1}{4}$ D) $\frac{1}{2}$

18. If $x = a\ cos^3\ \theta, y = a\ sin^3\ \theta, then$ $\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is equal to
   $A) tan^2\ \theta$ B) $sec^2\ \theta$ C) $\sec\theta$ D) $|\sec\theta|$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) **(A) is true but (R) is false.**
(d) **(A) is false but (R) is true.**

19. Assertion(A): For any symmetric matrix A, $B'AB$ is a skew-symmetric matrix.
Reason(R): A square matrix P is skew-symmetric, if $P' = -P$.
20. Assertion(A): Minor of an element of a determinant of order n(n≥2) is a determinant of order n.
Reason(R): If A is an invertible matrix of order 2, then det(A^{-1}) is equal to 1/det A.

Section-B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21. (a) Find the principal value of $\cos^{-1}\left\{\sin\left(\cos^{-1}\frac{1}{2}\right)\right\}$

OR

(b) Which is greater $\tan 1$ *or* $\tan^{-1} 1$? Justify your answer.

22. (a) If $\sin y = x\sin(a+y)$, *prove that* $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

OR

(b) Differentiate the function $y = (\sin x)^{\log x}$ with respect to x.

23. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then find the value(s) of θ satisfying the equation $A^T + A = I_2$.

24. Examine continuity of the function $f(x) = \begin{cases} \frac{\cos t}{\frac{\pi}{2}-t}, t \neq \frac{\pi}{2} \\ 1, t = \frac{\pi}{2} \end{cases}$ at $t = \frac{\pi}{2}$.

25. Find the equation of the line joining the points A(3,1) and B(9,3) using determinants.

Section-C

[This section comprises of short answer type questions (SA) of 3 marks each]

26. Let N be the set of all natural numbers and R be the relation on $N \times N$ defined by
$(a,b)R(c,d)\ iff\ a+d = b+c\ for\ all\ (a,b),(c,d) \in N \times N$. Show that R is an equivalence relation.

27. (a)Differentiate $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}$ with respect to $\cos^{-1} x^2$.

OR

(b) If $y = \log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$, *show that* $\frac{dy}{dx} - \sec x = 0$.

28. Express the matrix $\begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrix.

29. Find the values of a and b for which the function f defined by

$$f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x}, x < \frac{\pi}{2} \\ a \qquad , x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi-2x)^2}, x > \frac{\pi}{2} \end{cases}$$

is continuous at $x = \frac{\pi}{2}$.

30. (a) If $y = x^n\{a\cos(\log x) + b\sin(\log x)\}$, prove that $x^2\frac{d^2y}{dx^2} + (1-2n)\frac{dy}{dx} + (1+n^2)y = 0$.

OR

(b) If $x = a(1+\cos\theta)$, $y = a(\theta + \sin\theta)$, prove that $\frac{d^2y}{dx^2} = \frac{-1}{a}$ at $\theta = \frac{\pi}{2}$.

31. (a) For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ show that $A^2 - 5A + 4I = O$. Hence find A^{-1}.

OR

(b) Find the inverse of the matrix $A = \begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$ and show that $aA^{-1} = (a^2 + bc + 1)I - aA$.

Section-D

[This section comprises of long answer type questions (LA) of 5 marks each]

32. (a) A toy rocket is fired, from a platform, vertically into the air, its height above the ground after t seconds is given by $s(t) = at^2 + bt + c$, where $a, b, c \in R$; $a \neq 0$ and $s(t)$ is measured in metres. After 10 second, the rocket is 16 m above the ground; after 20 seconds, 22 m; after 30 seconds, 25 m.

(i) Write down a system of three linear equations in terms of a, b and c.

(ii) Hence find the values of a,b and c using matrix method.

OR

(b) If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are two square matrices, find AB and hence solve the system of linear equations: $x - y = 3$, $2x + 3y + 4z = 17$, $y + 2z = 7$.

33. If $f(x) = \begin{cases} x^2 + 3x + a, & for\ x \leq 1 \\ bx + 2 & , for\ x > 1 \end{cases}$ is everywhere differentiable, find the values of a and b.

34. (a)Let $A = \{x \in Z : 0 \leq x \leq 12\}$. Show that the relation $R = \{(a,b) : a, b \in A, |a-b| is\ divisible\ by\ 4\}$ is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2].

OR

(b) Let $A = R - \{3\}, B = R - \{1\}$. Let $f : A \to B$ be defined by $f(x) = \frac{x-2}{x-3}$, $\forall\, x \in A$. Show that f is bijective.

35. Find the adjoint of the matrix $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence show that $A(adj\ A) = |A|I_3$.

Section-E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.]

36. **Case Study-1:** A pharmaceutical company is developing a new medicine. The effectiveness of the medicine over time is modeled by a function f(t), where t is time in hours after the dose is taken. The company ensures that the concentration of the drug in the bloodstream is continuous over time for proper efficacy.

The function is given by:

$$f(t) = \begin{cases} 2t + 3, 0 \le t < 2 \\ at^2 + bt, 2 \le t \le 4 \\ 3t - 1, 4 < t \le 6 \end{cases}$$

To ensure safe and effective drug levels, the company wants the function f(t) to be continuous at t=2 and t=4.

Based on the information given above, answer the following questions:

(i) What condition must be satisfied for f(t) to be continuous at t=2? 1

(ii) Find the value of f(2) in terms of a and b. 1

(iii) (a) Find the values of a and b such that f(t) is continuous at t=2 and t=4. 2

OR

(b) Considering $a = -\frac{3}{8}, b = \frac{17}{4}, evaluate: \frac{f(4)-f(2)}{4-2}$. 2

37. **Case Study-2:**

A school is organizing its annual cultural fest. The event has three major segments: Music, Dance, and Drama. The school has a total of ₹50,000 to be spent on these three events. The following conditions are set by the organizing committee:

- The cost allocated to Dance is ₹5,000 more than Music.
- The Drama event requires twice as much funding as the Dance event.

Let the amount allocated to Music, Dance, and Drama be ₹x, ₹y, and ₹z respectively.

Based on the information given above, answer the following questions:

(i) Write the system of equations representing the above situation. 1

(ii) Represent the system of equations in the form **AX = B**, where **A** is the coefficient matrix, **X** the variable matrix, and **B** the constant matrix. 1

(iii) (a) If A is the coefficient matrix , find adj A. 2

OR

(b) Does the system of equations in question (i) consistent? Justify your answer. 2

38. **Case Study-3:**

A function f(x) is said to be differentiable at x=c if

(i) Left hand derivative (L.H.D.)= $\lim_{h \to 0^-} \frac{f(c+h)-f(c)}{h}$ exists finitely

(ii) Right hand derivative (R.H.D.)= $\lim_{h \to 0^+} \frac{f(c+h)-f(c)}{h}$ exists finitely and

(iii) L.H.D.=R.H.D. i.e., if function f(x) is differentiable at x=c, then $f'(c) = \lim_{x \to c} \frac{f(x)-f(c)}{x-c}$.

Based on the information given above, answer the following questions:

(i) $If\ f(2) = 4\ and\ f'(2) = 4$, then find the value of $\lim_{x \to 2} \frac{xf(2)-2f(x)}{x-2}$.

(ii) $If\ f(4) = 2, f'(4) = 3, g(4) = -2\ and\ g'(4) = 4$, then find the value of $\lim_{x \to 4} \frac{g(x)f(4)-g(4)f(x)}{x-4}$