07.07.25

Delhi Public School, Howrah

Periodic Test - I Examination (2025 - 2026)

Class - XII

Care must be taken not to write anything on the question paper. All the questions must be attempted in the correct sequence.

Subject: - Applied Mathematics (Code No- 241)

Time: 3 Hours **F.M. 80**

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This question paper contains **38** questions. **All** questions are **compulsory**.

(ii) This question paper is divided into 5 Sections – Section **A, B, C, D** and **E**.

(iii) In Section – **A**, Questions Number **1** to **18** are Multiple Choice Questions (MCQs) and Questions Number **19 and 20** are Assertion – Reason based questions of **1** mark each.

(iv) In Section – **B**, Questions Number **21** to **25** are Very Short Answer (VSA) type Questions, carrying **2** marks each.

(v) In Section – **C,** Questions Number **26** to **31** are Short Answer (SA) type Questions, carrying **3** marks each.

(vi) In Section – **D**, Questions Number **32** to **35** are Long Answer (LA) type Questions, carrying **5** marks each.

(vi) In Section – **E**, Questions Number **36** to **38** are case study-based questions, carrying **4** marks each.

(vii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **2** questions in Section – **C**, **2** questions in Section **D** and **3** questions in Section – **E**.

(viii) Use of calculators is **NOT** allowed.

SECTION - A

This section comprises of Multiple Choice Questions (MCQs) of 1 mark each. Select the correct option.

1) If x, y, z are positive real numbers, then the least value of $(x+y)(y+z)(z+x)$ is 1

(A) $8xyz$ (B) $2xyz$ (C) $64xyz$ (D) xyz

2) $(137+995)(mod\ 12)$=? 1

(A) 5 (B) 4 (C) 1 (D) 0

3) The rate of change of the area of a circle with respect to its radius when the radius is 6 cm is 1

(A) 12 π (B) 6 π (C) 18 π (D) 8 π

4) If $A = \begin{bmatrix} 0 & a & 1 \\ -1 & b & 1 \\ -1 & c & 0 \end{bmatrix}$ is a skew – symmetric matrix, then the values of $(a+b+c)^2$ is 1

(A) 1 (B) 0 (C) 4 (D) None of these

5) A point on the curve $y^2 = 18x$ at which the ordinate increases at twice the rate of abscissa is 1

(A) (2,4) (B) $(2,-4)$ (C) $(\frac{-9}{8}, \frac{9}{2})$ (D) $(\frac{9}{8}, \frac{9}{2})$

6) The value of $\begin{vmatrix} a-b & b+c & a \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix}$ is 1

(A) $a^3+b^3+c^3$ (B) $a^3+b^3+c^3-3abc$ (C) abc (D) $3abc$

7) The number of all possible matrices of order 3 x 3 with each entry 2 or 3 is 1

(A) 18 (B) 27 (C) 81 (D) None of these

8) If $f(x) = \log x$,then the derivative of $f(\log x)\ w.r.t\ x$ is 1

(A) $\frac{\log x}{x}$ (B) $\frac{x}{\log x}$ (C) $x \log x$ (D) $\frac{1}{x \log x}$

9) If $\begin{bmatrix} 1 & 3 & 9 \\ 1 & x & x^2 \\ 4 & 6 & 9 \end{bmatrix}$ is singular matrix, then $x =$? 1

(A) 3 (B) 3 or 6 (C) 3 or $\frac{3}{2}$ (D) $-3, \frac{3}{2}$

10) If $y = Ae^{4x} + Be^{-4x}$, then $\frac{d^2y}{dx^2}$ is equal to 1

(A) $16y$ (B) $4y$ (C) $-16y$ (D) $-4y$

11) The least value of the function $f(x) = ax + \frac{b}{x} (x > 0, a > 0, b > 0)$ is 1

(A) $\sqrt{ab}$ (B) $2\sqrt{ab}$ (C) ab (D) $2ab$

12) The function $f(x) = x^2 e^{-x}$ strictly increases on 1

(A) [0,2] (B) [0,∞) (C) $(-\infty, 0] \cup [2, \infty)$ (D) (2, ∞)

13) 20 litres of a mixture contains milk and water in the ratio 3:1. The amount of milk, in litres to be added to the mixture so as to have milk and water in the ratio 4:1 is 1

(A) 7 (B) 4 (C) 5 (D) 6

14) The matrix $\begin{bmatrix} 0 & -5 & 3 \\ 5 & 0 & -7 \\ -3 & 7 & 0 \end{bmatrix}$ is a 1

(A) diagonal matrix (B) symmetric matrix

(C) skew – symmetric matrix (D) scalar matrix

15) The radius of a cylinder is increasing at the rate of 3 cm/sec and its height is decreasing at the rate of 4 cm/sec. The rate of change of its volume (in cm^3/sec) when radius is 4 cm and height 6 cm is 1

(A) 64 (B) 144 (C) 80 (D) 80π

16) If the selling price of a commodity is fixed at ₹ 45 and the cost function is $C(x) = 30x + 240$, then the break - even point is 1

(A) $x = 10$ (B) $x = 12$ (C) $x = 15$ (D) $x = 16$

17) The smallest value of the polynomial $x^3 - 18x^2 + 96x$ in [0,9] is 1

(A) 126 (B) 135 (C) 160 (D) 0

18) Find all real values of x which satisfy the inequality : $x^3(x-1)(x-2) > 0$. 1

(A) $(0,1) \cup (2, \infty)$ (B) $(2, \infty)$ (C) $[0,1]$ (d) None of these

Questions number **19** and **20** are Assertion – Reason based questions of 1 mark each. Two statements are given – one labelled Assertion (A) and other labelled Reason ®. Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below:

(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true and R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false and R is true.

19) **Assertion(A):** The real value of k for which the system of linear equations $x - 2y = kz, z - ky = 2x$ and $y - 2z = kx$ has a non – zero solution is (-1).

Reason(R): The system of linear equations $a_1x + b_1y + c_1z = 0, a_2x + b_2y + c_2z = 0,\ a_3x + b_3y + c_3z = 0$ has a zero solution if $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$ 1

20) **Assertion(A):** If the average cost (AC) of producing x units of an item is given by $AC = 3x^2 - 7x + 5 - \frac{11}{x}$, then the marginal cost (MC) of producing 5 units is ₹ 160. 1

Reason(R): $\text{Marginal Cost} = \frac{\text{d}}{\text{dx}}(\text{Cost})$.

SECTION – B

Questions Number 21 to 25 are Very Short Answer (VSA) type questions of **2** marks each.

21) (a) If x, y, z are all different and $\begin{vmatrix} x & x^2 & x^3+1 \\ y & y^2 & y^3+1 \\ z & z^2 & z^3+1 \end{vmatrix} = 0$, then show that $xyz = -1$. 2

OR

(b) Find the matrix X for which $\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$. 2

22) If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$. 2

23) Solve the following inequality: $|5x| < |10 - 5x|$. 2

24) The fixed cost of a new product is ₹ 18000 and the variable cost per unit is ₹ 550. If the demand function is $p(x) = 4000 - 150x$, find the breakeven values. 2

25) (a) Find the last two digits of the product 567×123. 2

OR

(b) Find the last digit of 13^{13}. 2

SECTION - C

Questions Number 26 to 31 are Short Answer (SA) type questions of **3** marks each.

26) A man wants to cut three lengths from a single piece of cardboard of length 91 cm. The second length is to be 3 cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths for the shortest board if the third piece is to be at least 5 cm longer than the second? 3

27) Show that the local maximum value of $x + \frac{1}{x}$ is less than the local minimum value. 3

28) (a) If A, B are skew-symmetric matrices of same order and AB is symmetric, then show that $AB = BA$. 3

OR

(a) Express the matrix A as the sum of a symmetric and a skew-symmetric matrices. 3

$$A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$$

29) (a) Solve the following system of linear equations using Cramer's Rule: 3

$$x + y + z = 1,\ ax + by + cz = k,\ a^2x + b^2y + c^2z = k^2.$$

OR

(b) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & x \\ -2 & 2 & -1 \end{bmatrix}$ is a matrix satisfying $AA' = 9I_3$, then find x. 3

30) The demand function of a monopolist is $p(x) = 300 - x^2$. Evaluate: 3

(a) Revenue Function

(b) Marginal Revenue Function

(c) Output x where revenue is maximum.

31) A tour operator charges ₹ 136 per passenger for 100 passengers with a discount of ₹ 4 for each 10 passengers in excess of 100. Find the number of passengers that will maximize the amount of money the tour operator receives. 3

SECTION - D

Questions Number **32** to **35** are Long Answer (LA) type questions of **5** marks each.

32) (a) An open box with a square base is to be made out of a given quantity of a cardboard of area c^2.Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$. 5

OR

(b) For a monopolist's product, the demand function is $p = \frac{50}{\sqrt{x}}$ and average cost function $AC = 0.5 + \frac{2000}{x}$. Find the profit maximizing level of output. At this level, show that the marginal revenue and marginal cost are equal. 5

33) Find the remainder when $862 \times 783 \times 671 \times 549 \times 411 \times 395 \times 297$ is divided by 7. 5

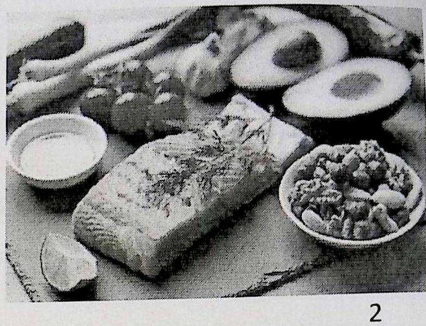
34) (a) Find the integral value of x if $[x \quad -4 \quad 1] \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 0 \\ 2 & 2 & 4 \end{bmatrix} [x \quad 4 \quad -1]^T = O$. 5

OR

(b) (i) If A is a matrix such that $A^3 = I$, then show that A is invertible. 3

(ii) If $A = \begin{bmatrix} -2 & 1 \\ 3 & 4 \end{bmatrix}$, find $A(adj\ A)$. 2

35) A diet is to contain 30 units of Vitamin A, 40 units of Vitamin B and 20 units of Vitamin C. Three types of foods F_1, F_2, F_3 are available. One unit of Food F_1 contains 3 units of Vitamin A, 2 units of Vitamin B and 1 unit of Vitamin C. One unit of food F_2 contains 1 unit of Vitamin A, 2 units of Vitamin B and 1 unit of vitamin C. One unit of food F_3 contains 5 units of Vitamin A, 3 units of Vitamin B and 2 units of Vitamin C. Based on the above information, answer the following questions:



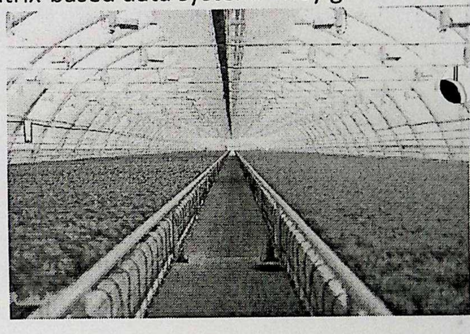
(a) If the diet contains x units of food F_1, y units of Food F_2 and z units of Food F_3, write the matrix equation representing the above situation. 2

(b) What are the values of x, y, z in given situation? 3

SECTION – E

Questions Number **36** to **38** are case – study based questions of **4** marks each.

36) A group of agri-science students have started a smart greenhouse project called "GreenGrid". Their aim is to monitor and optimize the nutrient supply for different plants using a matrix-based data system. They grow three types of plants: Lettuce (Plant A), Bell Pepper (Plant B) and Tomato (Plant C). Each plant requires a specific quantity of three nutrients: Nitrogen (N), Phosphorus (P), and Potassium (K), measured in grams per plant per week. Their weekly nutrient requirement matrix is:



$\begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 2 \\ 3 & 2 & 1 \end{bmatrix}$ where columns represent Plants A, B, and C, and rows represent nutrients N, P, K respectively. Now they want to supply nutrients in such a way that the total consumption across all plants per week is: $\begin{bmatrix} 100 \\ 90 \\ 80 \end{bmatrix}$ (grams of N, P, K respectively). They want to know how many plants of each type (A, B, and C) they can grow without exceeding the weekly nutrient limits. Based on the above information, answer the following questions.

(a) Represent the system of equations derived from the given data. 1

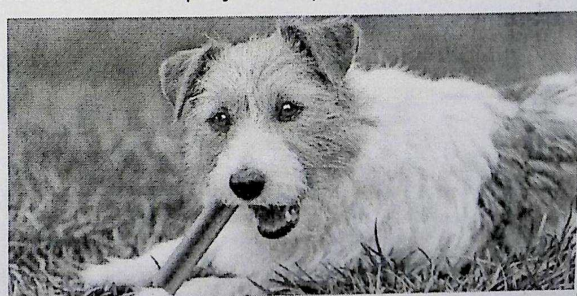
(b) Find the determinant of the coefficient matrix. 1

(c) If a student suggests growing 10 lettuces, 20 bell peppers and 10 tomatoes, verify whether this is within the nutrient limits. 2

OR

(d) How many of each plant can they grow? 2

37) Rohit and Ananya are passionate pet lovers who recently launched their dream project – a premium dog biscuit brand called "Biscuit & Bark." Their aim is to create healthy, tasty treats for dogs across the city. They set up a small production unit and started manufacturing the biscuits in bulk. From the beginning, they noticed a non-linear cost pattern: as production increases, the cost per kilogram fluctuates due to raw materials, packaging, and machine wear. After studying the trend, they found that the total cost of producing x kg of biscuits per day could be modelled by the function: $C(x) = 5x^3 - 60x^2 + 300x + 1000$ (Where C(x) is the cost in ₹ and x is in kg). They sell the biscuits for ₹ 150 per kg, so their revenue function becomes: $R(x) = 150x$. Now, Rohit wonders, "At what production level will we maximize our profit?" So, they consult their friend Neha, a math enthusiast, who uses derivatives to answer their business questions. Based on the above information,answer the followig questions:



(a) Find the profit function. 1

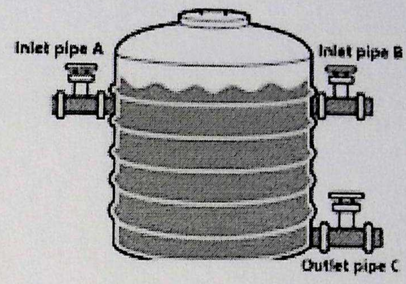
(b) Is the profit increasing or decreasing at $x = 10$? Justify with derivative. 1

(c) Find the critical points of the profit function. [Use $\sqrt{6} = 2.45$] 2

OR

(d) Determine the maximum profit value. 2

38) For providing water to the families of a colony, a large water tank with two inlet pipes A and B and an outlet pipe C, is installed. Pipes A and B can fill the tank in 10 hours and 12 hours respectively; whereas pipe C can empty the tank in 15 hours. Based on the above information, answer the following questions:



(i) Pipe A is opened alone for 2 hours. Then pipes B and C are opened together along with A. In how many more hours will the tank be completely filled? 1

(ii) If both pipes A and C are opened together, then find the time in which the tank will be filled completely. 1

(iii) (a) If the efficiency of pipe A is increased by 50% and that of pipe B is decreased by 25%, in how much time will the tank be filled when both are opened together? 2

OR

(b) Pipes A and B are opened together for some time and then pipe B is turned off after some time. If the tank is completely filled in 6 hours, then after how many hours is pipe B turned off? 2

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