Delhi Public School, Howrah

PERIODIC TEST-II (2025-2026)

Class-XII

Care must be taken not to write anything on the question paper. All the questions must be attempted in the correct sequence.

Subject: - Mathematics (Code No-041)

Time: 3 Hours **F.M. 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.

3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.

4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.

5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.

6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

Section- A

(Multiple Choice Questions)

Each question carries 1 mark

1. The function $f:[0,\infty)\to R$ given by $f(x)=\frac{x}{x+1}$ is

   a) One-one and onto b) one-one but not onto c)onto but not one-one d) neither one-one nor onto

2. Which of the following corresponds to the principal value branch of $\tan^{-1}x$?

   a) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ b) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ c) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)-\{0\}$ d) $(0,\pi)$

3. If $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix}=\begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then the value of $\left(\frac{24}{x}+\frac{24}{y}\right)$ is

   a) 7 b) 6 c) 8 d) 18

4. If $A=\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then the value of |adj A| is

   a) a^{27} b) a^9 c) a^6 d) a^2

5. If the function $f(x)=\begin{cases} \frac{\sin 3x}{x}, x\neq 0 \\ \frac{k}{2} \quad, x=0 \end{cases}$ is continuous at x=0, then k=?

   a) 3 b)6 c)9 d) 12

6. Which of the following statements is true for the function $f(x)=\begin{cases} x^2+3, x\neq 0 \\ 1 \quad, x=0 \end{cases}$?

   a) f(x) is continuous and differentiable $\forall x\in R$.
   b) f(x) is continuous $\forall x\in R$.
   c) f(x) is continuous and differentiable $\forall x\in R-\{0\}$
   d) f(x) is discontinuous at infinitely many points

7. If the function $f(x)=2x^2-kx+5$ is increasing on [1,2], then k lies in the interval

   a) $(-\infty,4)$ b) $(4,\infty)$ c) $(-\infty,8)$ d) $(8,\infty)$

8. If $f(x)=\frac{1}{4x^2+2x+1}$, then its maximum value is

   a) $\frac{4}{3}$ b) $\frac{2}{3}$ c) 1 d) $\frac{3}{4}$

9. $\int \frac{f'(x)}{f(x)\log(f(x))}\, dx$ is equal to

a) $f(x)\log(f(x)) + C$ b)$\log(\log(f(x))) + C$ c) $\frac{f(x)}{\log(f(x))} + C$ d) $\frac{1}{\log(\log(f(x)))} + C$

10. If $\int_0^k \frac{1}{9x^2+1} dx = \frac{\pi}{12}$, then k is equal to

a) $\frac{\pi}{4}$ b) $\frac{1}{3}$ c) 3 d) none of these

11. The number of corner points of the feasible region determined by the constraints: $x \geq 0, y \geq 0, x + y \geq 4$ is

a) 0 b) 1 c) 2 d) 3

12. If the matrix $\begin{bmatrix} 3 & 2x & -5 \\ -4 & 0 & y \\ -5 & 3 & 7 \end{bmatrix}$ is symmetric, then which of the following is true?

a) x=2,y=-3 b) x=2,y=3 c) x=-2,y=-3 d) x=-2,y=3

13. If $x = a\, cos^3\theta, y = a\, sin^3\theta, then\ \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$=?

a) $tan^2\theta$ b) $sec^2\theta$ c) $\sec\theta$ d)$|\sec\theta|$

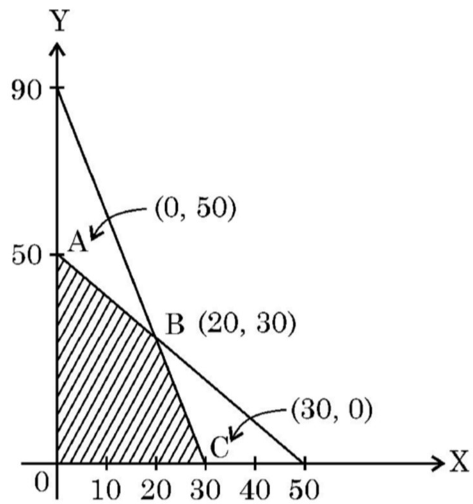
14. The volume of a sphere is increasing at the rate of $4\pi\ cm^3/sec$. The rate of increase of the radius when the volume is $288\pi\ cm^3$, is

a) $\frac{1}{4}$ b) $\frac{1}{12}$ c) $\frac{1}{36}$ d) $\frac{1}{9}$

15. If the area bounded by the curve $y^2 = 16x$ and the line $y = mx$ is $\frac{2}{3}$ sq. units, then the value of m is

a) 1 b) 2 c) 3 d) 4

16. The maximum value of $Z = 4x + y$ for a L.P.P. whose feasible region is given below is:



a) 50 b)110 c)120 d)170

17. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{1+\cos 2x}$ is equal to

a) 1 b)2 c)3 d)4

18. If R is a relation on N defined by $nRm\ iff\ n\ divides\ m$, then R is

a) reflexive and symmetric b) transitive and symmetric c) reflexive and transitive d) equivalence relation

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

19. **Assertion(A):** Let $f: R \rightarrow R$ be a function such that $f(x) = x^3 + x^2 + 3x + \sin x$. Then f is an increasing function.

    Reason (R): If $f'(x) < 0$, then f(x) is a decreasing function.

20. **Assertion(A):** For matrix $A = \begin{bmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{bmatrix}, where\ \theta \in [0,2\pi], |A| \in [2,4]$.

    Reason(R): $\cos\theta \in [-1,1], \forall\theta \in [0,2\pi]$.

Section-B

[**This section comprises of very short answer type questions (VSA) of 2 marks each**]

21. Find the principal value of $\tan^{-1} 1 + \cos^{-1}\left(-\frac{1}{2}\right)$

22. If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}, show\ that\ A^2 = 4A - 3I$, where I is the identity matrix of order 2. Hence find A^{-1}.

23. (a) Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$, if $x = ae^{\theta}(\sin\theta - \cos\theta)\ and\ y = ae^{\theta}(\sin\theta + \cos\theta)$.

    OR

    (b) Find $\frac{dy}{dx}$ $, if\ y = x^{\tan x} + \frac{\sqrt{x^2+1}}{2}$.

24. Determine whether the function $f(x) = \cos\left(2x + \frac{\pi}{4}\right)$ is increasing or decreasing in the interval $\left(\frac{3\pi}{8}, \frac{7\pi}{8}\right)$.

25. (a) If $\int \frac{2^{\frac{1}{x}}}{x^2} dx = k.2^{\frac{1}{x}} + C$, then find the value of k.

    OR

    (b) Evaluate: $\int_0^{\pi} \frac{\sin 2px}{\sin x} dx, p \in N$.

Section-C

[This section comprises of short answer type questions (SA) of 3 marks each]

26. Solve the following linear programming problem graphically:

    Maximize $Z = 3x + 9y$, subject to the constraints $x + y \geq 10, x + 3y \leq 60, x \leq y, x \geq 0, y \geq 0$.

27. (a) If $y = \left(x + \sqrt{1 + x^2}\right)^n, prove\ that\ (1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2y$.

    OR

    (b) If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y), then\ prove\ that\ \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

28. If the function $f(x) = 2x^3 - 9mx^2 + 12m^2x + 1, where\ m > 0$ attains its maximum and minimum at $p\ and\ q$ respectively such that $p^2 = q$, then find the value of m.

29. (a) Evaluate: $\int \frac{(3\sin\theta-2)\cos\theta}{5-cos^2\theta-4\sin\theta}\,d\theta$

OR

(b) Evaluate: $\int_0^{\pi} e^{2x}\sin\left(\frac{\pi}{4}+x\right)dx$

30. The area of the region bounded by the line $y = mx(m > 0)$, $the\ curve\ x^2 + y^2 = 4$ and the x-axis in the first quadrant is $\frac{\pi}{2}$ units. Using integration, find the value of m.

31. Find a and b, if the function f(x) given by $f(x) = \begin{cases} ax^2 + b, if\ x < 1 \\ 2x + 1, if\ x \geq 1 \end{cases}$ is differentiable at x=1.

Section-D

[This section comprises of long answer type questions (LA) of 5 marks each]

32. (a) A school wants to allocate students into three clubs: Sports, Music and Drama, under the following conditions:
    - The number of students in Sports club should be equal to the sum of the number of students in Music and Drama club.
    - The number of students in Music club should be 20 more than half the number of students in Sports club.
    - The total number of students to be allocated in all three clubs are 180.

    Find the number of students allocated to different clubs, using matrix method.

OR

(b) If $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ $and\ B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$ are two square matrices, find AB and hence solve the system of linear equations: $x - 2y = 3$, $2x - y - z = 2, -2y + z = 3.$

33. (a) Prove that the relation R on the set $N \times N$ defined by $(a, b)R(c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in N \times N$ is an equivalence relation. Find the equivalence classes [(2,3)] and [(1,3)].

OR

(b) Show that the function $f: R \to R$ defined as $f(x) = x^2 + x + 1$ is neither one-one nor onto. Also, find all the values of x for which f(x)=3.

34. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also find the maximum volume of cone.

35. Find the values of $p\ and\ q$, for which the function f(x) given by

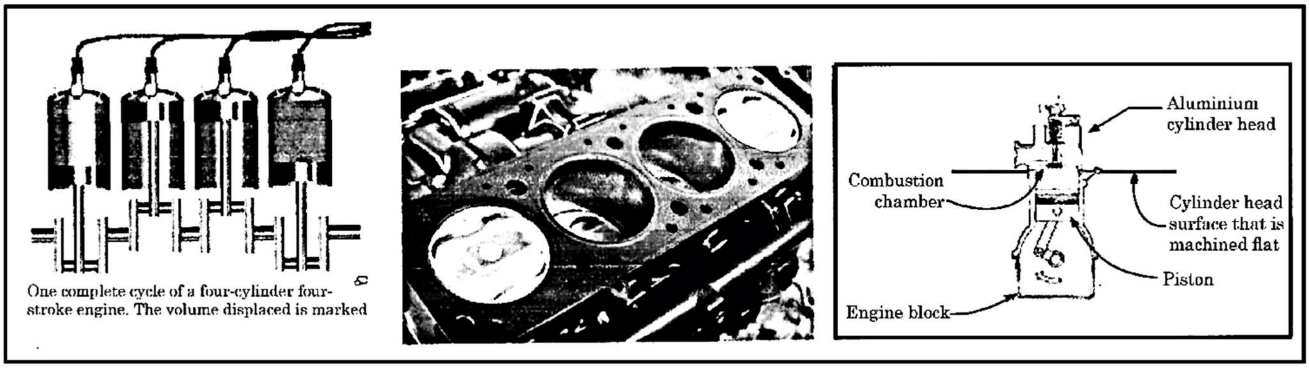
$$f(x) = \begin{cases} \dfrac{1 - sin^3x}{3cos^2x}, if\ x < \dfrac{\pi}{2} \\ p \qquad\qquad , if\ x = \dfrac{\pi}{2} \\ \dfrac{q(1 - \sin x)}{(\pi - 2x)^2}, if\ x > \dfrac{\pi}{2} \end{cases}$$

is continuous at $x = \frac{\pi}{2}$.

Section-E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.]

36. **Case Study-1:**
Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore.



The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area 75π cm^2.

Based on the information given above, answer the following questions:

(i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r. 1

(ii) Find $\frac{dV}{dr}$ 1

(iii) (a) Find the radius of cylinder when its volume is maximum. 2

OR

(b) For maximum volume, h>r. State true or false and justify. 2

37. **Case Study-2:** Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹160. From the same shop, Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹250.

Based on the above information, answer the following questions :

(i) Convert the given above situation into a matrix equation of the form **AX = B**. 1

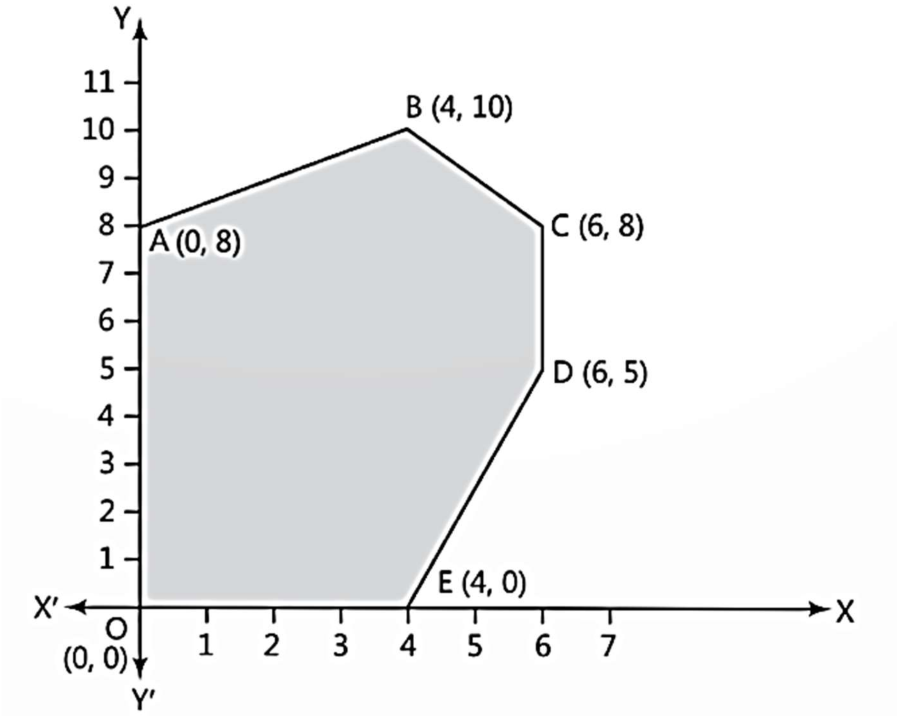
(ii) Find |A|. 1

(iii) (a) Find A^{-1}. 2

OR

(b) Determine $P=A^2-5A$ 2

38. **Case Study-3:** The corner points of the feasible region of an LPP determined by the system of linear constraints are as shown below:



Based on the information given above, answer the following questions:

(i) Let $z = 3x - 4y$ be the objective function. Find the maximum and minimum value of Z and also the corresponding points at which the maximum and minimum value occurs. 2

(ii) Let $Z = px + qy, where\ p, q > 0$ be the objective function. Find the condition on $p\ and\ q$ so that the maximum value of Z occurs at B(4,10) and C(6,8). Also mention the number of optimal solutions in this case. 2