Delhi Public School, Howrah

PERIODIC TEST-III (2025-2026)

Class-XII

Care must be taken not to write anything on the question paper. All the questions must be attempted in the correct sequence.

Subject: - Mathematics (Code No-041)

Time: 3 Hours **F.M. 80**

General Instructions:

(i) This Question paper contains 38 questions. All questions are compulsory.
(ii) This Question paper is divided into five Sections - A, B, C, D and E.
(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
(ix) Use of calculators is not allowed.

Section- A
(Multiple Choice Questions)
Each question carries 1 mark

1. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ and $A^2 = 3I$, then which of the following is correct?
   a) $1 + \alpha^2 + \beta\gamma = 0$ b) $1 - \alpha^2 - \beta\gamma = 0$ c) $3 - \alpha^2 - \beta\gamma = 0$ d) $3 + \alpha^2 + \beta\gamma = 0$
2. If A is a square matrix of order 2 and $|A| = -2$, then the value of $|5A^t|$ is
   a) -50 b) -10 c) 10 d)50
3. If the area of the triangle with vertices (-3,0),(3,0) and (0,k) is 9 sq. units, then the value(s) of k will be
   a) 9 b) ±3 c)-9 d) 6
4. For what value of k, the function given below is continuous at x=0?
$$f(x) = \begin{cases} \dfrac{\sqrt{4+x}-2}{x}, x \neq 0 \\ k, x = 0 \end{cases}$$
   a) 0 b) 1/4 c) 1 d) 4
5. The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{4-z}{k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{-2}$ are mutually perpendicular, if the value of k is
   a) -2/3 b) 2/3 c) -2 d) 2
6. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ is
   a) 3 b) 2 c) 1 d) not defined
7. The objective function $Z = ax + by$ of an LPP has maximum value 42 at (4,6) and minimum value 19 at (3,2). Which of the following is true?
   a) a=9,b=1 b) a=5,b=2 c) a=3,b=5 d) a=5,b=3
8. If $\left|\vec{a} \times \vec{b}\right| = \sqrt{3}$ and $\vec{a}.\vec{b} = -3$, then the angle between $\vec{a}$ and $\vec{b}$ is
   a) $\frac{2\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{3}$ d) $\frac{5\pi}{6}$

9. $\int_{-1}^{1} |1-x| dx$ is equal to
   a) -2 b) 2 c)0 d) 4
10. Given that $A = [a_{ij}]$ is a square matrix of order 3×3 and |A|=-7, then the value of $\sum_{i=1}^{3} a_{i2}A_{i2}$, whereA_{ij} denotes the cofactor of element a_{ij} , is
    a) 7 b) -7 c)0 d) 49
11. The region represented by the inequation system $x, y \geq 0, y \leq 6, x + y \leq 3$ is
    a) unbounded in first quadrant
    b) unbounded in first and second quadrant
    c) bounded in first quadrant
    d) bounded in second quadrant
12. The value of p for which the vectors $2\hat{\imath} + p\hat{\jmath} + \hat{k}\ and - 4\hat{\imath} - 6\hat{\jmath} + 26\hat{k}$ are perpendicular to each other, is
    a) 3 b)-3 c) $-\frac{17}{3}$ d) $\frac{17}{3}$
13. If $A = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 2 & 3 \\ 5 & 1 & -1 \end{bmatrix}$, then the value of |A(adj A| is
    a) $100I$ b) $10I$ c) 10 d) 1000
14. If the sum of numbers obtained on throwing a pair of dice is 9, then the probability that number obtained on one of the dice is 4, is
    a) 1/9 b) 4/9 c) 1/18 d) 1/2
15. The solution of the differential equation $\frac{dy}{dx} + \frac{2y}{x} = 0\ with\ y(1) = 1$ is given by
    a) $y = \frac{1}{x^2}$ b) $x = \frac{1}{y^2}$ c) $x = \frac{1}{y}$ d) $y = \frac{1}{x}$
16. If the projection of $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}\ on\ \vec{b} = 2\hat{\imath} + \lambda\hat{k}$ is zero, then the value of λ is
    a) 0 b) 1 c) -2/3 d) -3/2
17. If $f(x) = 2|x| + 3|\sin x| + 6$, then the right hand derivative of f(x) at x=0 is
    a) 6 b) 5 c) 3 d) 2
18. If a line makes angles $\alpha, \beta\ and\ \gamma$ with the coordinate axes respectively, then the value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ is
    a) -2 b) -1 c) 1 d) 2

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

19. **Assertion(A):** If x is real, then the minimum value of x^2-8x+17 is 1.
    Reason (R): If $f''(x) > 0$ at a critical point, then the value of the function at the critical point will be the minimum value of the function.
20. **Assertion(A):** A relation $R = \{(a, b): |a - b| < 3\}$ defined on the set $A = \{1,2,3,4\}$ is reflexive.
    Reason(R): A relation R on the set A is said to be reflexive if for $(a, b) \in R\ and\ (b, c) \in R$, $we\ have\ (a, c) \in R$.

Section-B

[**This section comprises of very short answer type questions (VSA) of 2 marks each**]

21. (a) Find the value of k, if $\sin^{-1}\left[k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]=\frac{\pi}{3}$

OR

(b) Find the domain of the function $y=\cos^{-1}(|x-1|)$.

22. Find the intervals in which $f(x)=(x-1)^3(x-2)^2$ is strictly increasing.

23. Find the maximum profit that a company can make, if the profit function is given by $P(x)=72+42x-x^2$, where x is the number of units and P is the profit in rupees.

24. (a) Evaluate: $\int_0^{\frac{\pi}{4}}\sqrt{1-\sin 2x}\ dx$

OR

(b) Evaluate: $\int_0^{\frac{\pi}{4}}\frac{tan^3 x}{1+\cos 2x}\ dx$

25. Find the points on the curve $6y=x^3+2$ at which ordinate is changing 8 times as fast as abscissa.

Section-C

[This section comprises of short answer type questions (SA) of 3 marks each]

26. Solve the following linear programming problem graphically:
Minimise $Z=600x+400y$, subject to the constraints $x+y\geq 8, x+2y\leq 16, 4x+y\leq 29, x,y\geq 0$.

27. (a)Evaluate: $\int\frac{2\cos x}{(1-\sin x)(2-cos^2 x)}\ dx$

OR

(b) Evaluate: $\int\frac{x^2+x+1}{(x+1)^2(x+2)}\ dx$

28. (a) Solve the differential equation $x^2y\ dx-(x^3+y^3)dy=0$

OR

(b) Solve the differential equation $(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$ subject to the condition $y(0)=0$.

29. Three events A,B and C have probabilities $\frac{2}{5}, \frac{1}{3}\ and\ \frac{1}{2}$ respectively. Given that $P(A\cap C)=\frac{1}{5}$ and $P(B\cap C)=\frac{1}{4}$, then find

(i) $P(C|B)$

(ii) $P(A'\cap C')$

(iii) $P(A'|C')$

30. (a) If $y=\left\{x+\sqrt{x^2+1}\right\}^m$, $show\ that\ (x^2+1)y_2+xy_1-m^2y=0$.

OR

(b) If $x=a\sin t-b\cos t, y=a\cos t+b\sin t$, then prove that $\frac{d^2y}{dx^2}=-\frac{(x^2+y^2)}{y^3}$.

31. Evaluate: $\int_0^{\frac{\pi}{2}}\frac{sin^2 x}{\sin x+\cos x}\ dx$

Section-D

[This section comprises of long answer type questions (LA) of 5 marks each]

32. (a) If $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & 0 & 1 \end{bmatrix}$, $find\ A^{-1}$. Hence, solve the following system of linear equations:
$x + 2y + z = 5, 2x + 3y = 1, x - y + z = 8.$

OR

(b) Use the product of the matrices $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ and hence solve the system of linear equations: $x + 2y - 3z = -4, 2x + 3y + 2z = 2, 3x - 3y - 4z = 11.$

33. Using integration, find the area of the region bounded by the lines $y = 4x + 5, x + y = 5\ and\ 4y = x + 5.$
34. Show that the relation R on the set $A = \{x \in Z: 0 \leq x \leq 12\}$, given by $R = \{(a, b): |a - b|\ is\ a\ multiple\ of\ 4\}$ is an equivalence relation. Find the set of all elements related to 1.
35. (a) Two vertices of the parallelogram ABCD are given as A(-1,2,1) and B(1,-2,5). If the equation of the line passing through C and D is $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$, then find the distance between the sides AB and CD. Hence, find the area of the parallelogram ABCD.

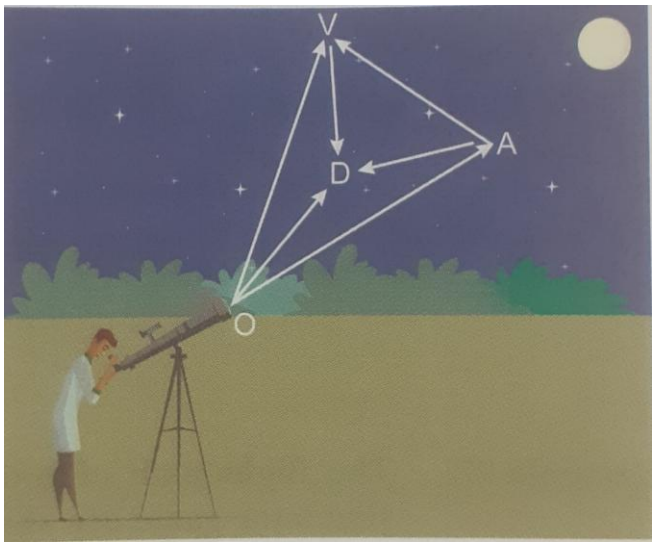
OR

(b) An aeroplane is flying along the line $\vec{r} = \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$, where λ is a scalar and another aeroplane is flying along the line $\vec{r} = \hat{\imath} - \hat{\jmath} + \mu(-2\hat{\jmath} + \hat{k})$, where μ is a scalar. At what points on the lines should they reach, so that the distance between them is the shortest? Find the shortest possible distance between them.

Section-E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.]

36. **Case Study-1:**
An instructor at the astronomical centre points out three of the brightest stars in a particular constellation. Assume that the telescope is located at O(0,0,0) and the three stars have their locations at the points D, A and V having position vectors $2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}, 7\hat{\imath} + 5\hat{\jmath} + 8\hat{k}\ and\ -3\hat{\imath} + 7\hat{\jmath} + 11\hat{k}$ respectively.



Based on the above information, answer the following questions:

(i) How far is the star V from star A? 1

(ii) Find a unit vector in the direction of $\overrightarrow{DA}$ 1

(iii) (a) Find the measure of $\angle VDA$ 2

OR

(b) What is the projection of $\overrightarrow{DV}$ on $\overrightarrow{DA}$. 2

37. **Case Study-2:**

In an elliptical sport field, the authority wants to design a rectangular soccer field with the maximum possible area. The sport field is given by the graph of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.



Based on the given information, answer the following questions:

(i) If the length and breadth of the rectangular field be 2x and 2y respectively, then find the area function in terms of x. 1

(ii) Find the critical point of the area function. 1

(iii) (a) Use first derivative test to find the length 2x and width 2y of the soccer field(in terms of a and b) that maximise its area. 2

OR

(b) Use second derivative test to find the length 2x and width 2y of the soccer field(in terms of a and b) that maximise its area. 2

38. **Case Study-3:**

An insurance company believes that people can be divided into two classes: those who are accident-prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20% of the population is accident-prone.



Based on the given information, answer the following questions:

(i) What is the probability that a new policyholder will have an accident within a year of purchasing a policy? 2

(ii) Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident-prone? 2